The pump handles liquid water which is incompressible, i.e. its density or specific volume undergoes little change with an increase in pressure. For reversible adiabatic compression, by the use of the general property relation

$$T ds = dh - v dp; \; ds = 0$$

and

$$dh = v dp$$

Since change in specific volume is negligible

$$\Delta h = v \Delta p$$

or

$$h_4 - h_3 = v_3 (p_1 - p_2)$$

If v is in m³/kg and p is in bar

$$h_4 - h_3 = v_3 (p_1 - p_2) \times 10^5 \text{ J/kg} \tag{12.7}$$

The *work ratio is* defined as the ratio of net work output to positive work output.

$$\therefore \quad \text{work ratio} = \frac{W_{net}}{W_T} = \frac{W_T - W_P}{W_T}$$

Usually, the pump work is quite small compared to the turbine work and is sometimes neglected. Then $h_4 = h_3$, and the cycle efficiency approximately becomes

$$\eta \cong \frac{h_1 - h_2}{h_1 - h_4}$$

The efficiency of the Rankine cycle is presented graphically in the T–s plot in Fig. 12.6. Thus Q_1 is proportional to area 1564, Q_2 is proportional to area 2563, and W_{net} (= $Q_1 - Q_2$) is proportional to area 1 2 3 4 enclosed by the cycle.

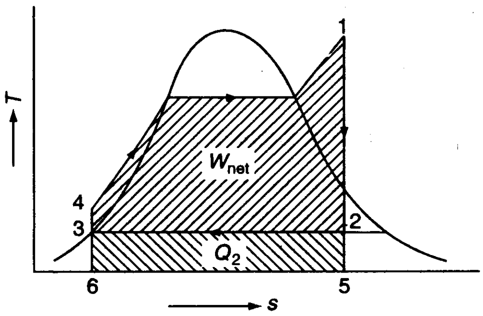


Fig. 12.6 Q_1, W_{net} and Q_2 are proportional to areas

The capacity of a steam plant is often expressed in terms of *steam rate*, which is defined as the rate of steam flow (kg/h) required to produce unit shaft output (1 kW). Therefore

$$\text{Steam rate} = \frac{1}{W_T - W_P} \frac{\text{kg}}{\text{kJ}} \cdot \frac{1 \text{ kJ/s}}{1 \text{ kW}}$$

$$= \frac{1}{W_T - W_P} \frac{\text{kg}}{\text{kWs}} = \frac{3600}{W_T - W_P} \frac{\text{kJ}}{\text{kWh}} \tag{12.8}$$

The cycle efficiency is sometimes expressed alternatively as heat rate which is the rate of heat input (Q_1) required to produce unit work output (1 kW)

$$\text{Heat rate} = \frac{3600 \, Q_1}{W_T - W_P} = \frac{3600}{\eta_{cycle}} \frac{\text{kJ}}{\text{kWh}} \tag{12.9}$$

From the equation $W_{rev} = -\int_1^2 v \, dp$, it is obvious that the reversible steady-flow work is closely associated with the specific volume of fluid flowing through the device. The larger the specific volume, the larger the reversible work produced or consumed by the steady-flow device. Therefore, every effort should be made to keep the specific volume of a fluid as small as possible during a compression process to minimize the work input and as large as possible, during an expansion process to maximize the work output.

In steam or gas power plants (Chapter 13), the pressure rise in the pump or compressor is equal to the pressure drop in the turbine if we neglect the pressure losses in various other components. In steam power plants, the pump handles liquid, which has a very small specific volume, and the turbine handles vapour, whose specific volume is many times larger. Therefore, the work output of the turbine is much larger than the work input to the pump. This is one of the reasons for the overwhelming popularity of steam power plants in electric power generation.

If we were to compress the steam exiting the turbine back to the turbine inlet pressure before cooling it first in the condenser in order to "save" the heat rejected, we would have to supply all the work produced by the turbine back to the compressor. In reality, the required work input would be still greater than the work output of the turbine because of the irreversibilities present in both processes (see Example 12.1).

12.3 ACTUAL VAPOUR CYCLE PROCESSES

The processes of an actual cycle differ from those of the ideal cycle. In the actual cycle conditions might be as indicated in Figs 12.7 and 12.8, showing the various losses. The thermal efficiency of the cycle is

$$\eta_{th} = \frac{W_{net}}{Q_1}$$

where the work and heat quantities are the measured values for the actual cycle, which are different from the corresponding quantities of the ideal cycle.

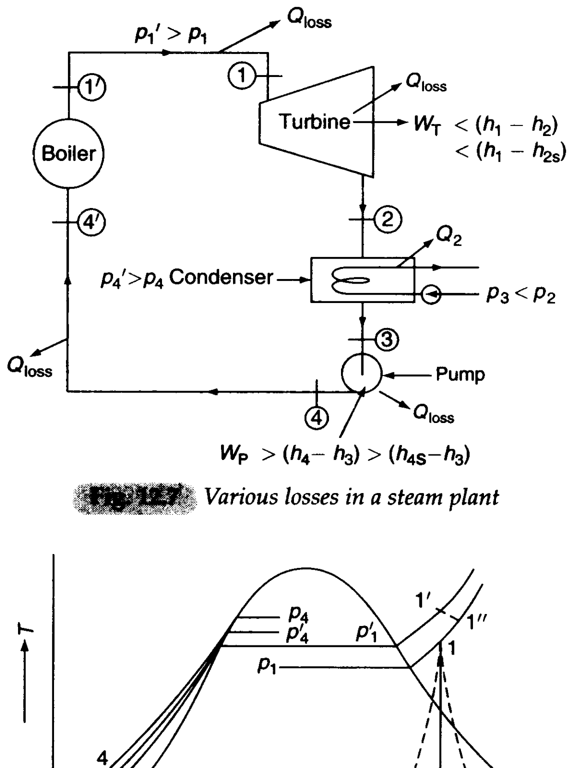


Fig. 12.7 Various losses in a steam plant

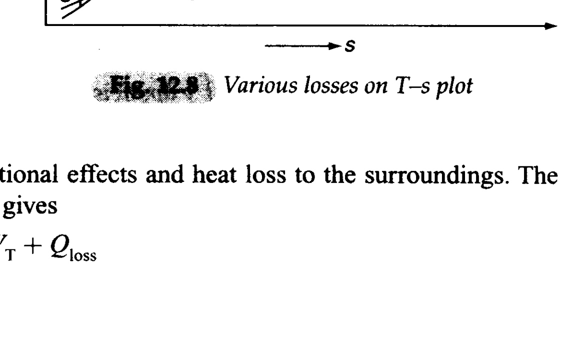


Fig. 12.8 Various losses on T–s plot

12.3.1 Piping Losses

Pressure drop due to friction and heat loss to the surroundings are the most important piping losses. States 1′ and 1 (Fig. 12.8) represent the states of the steam leaving the boiler and entering the turbine respectively, 1′ − 1″ represents the frictional losses, and 1″ − 1 shows the constant pressure heat loss to the surroundings. Both the pressure drop and heat transfer reduce the availability of steam entering the turbine.

A similar loss is the pressure drop in the boiler and also in the pipeline from the pump to the boiler. Due to this pressure drop, the water entering the boiler must be pumped to a much higher pressure than the desired steam pressure leaving the boiler, and this requires additional pump work.

12.3.2 Turbine Losses

The losses in the turbine are those associated with frictional effects and heat loss to the surroundings. The steady flow energy equation for the turbine in Fig. 12.7 gives

$$h_1 = h_2 + W_T + Q_{loss}$$

$$\therefore \qquad W_T = h_1 - h_2 - Q_{loss} \tag{12.10}$$

For the reversible adiabatic expansion, the path will be 1 – 2*s*. For an ordinary real turbine the heat loss is small, and W_T is $h_1 - h_2$, with Q_2 equal to zero. Since actual turbine work is less than the reversible ideal work output, h_2 is greater than h_{2s}. However, if there is heat loss to the surroundings, h_2 will decrease, accompanied by a decrease in entropy. If the heat loss is large, the end state of steam from the turbine may be 2′. It may so happen that the entropy increase due to frictional effects just balances the entropy decrease due to heat loss, with the result that the initial and final entropies of steam in the expansion process are equal, *but the expansion is neither adiabatic nor reversible*. Except for very small turbines, heat loss from turbines is generally negligible. The isentropic efficiency of the turbine is defined as

$$\eta_T = \frac{W_T}{h_1 - h_{2s}} = \frac{h_1 - h_2}{h_1 - h_{2s}} \tag{12.11}$$

where W_T is the actual turbine work, and $(h_1 - h_{2s})$ is the isentropic enthalpy drop in the turbine (i.e. the ideal output).

12.3.3 Pump Losses

The losses in the pump are similar to those of the turbine, and are primarily due to the irreversibilities associated with fluid friction. Heat transfer is usually negligible. The pump efficiency is defined as

$$\eta_P = \frac{h_{4s} - h_3}{W_P} = \frac{h_{4s} - h_3}{h_4 - h_3} \tag{12.12}$$

where W_P is the actual pump work.

12.3.4 Condenser Losses

The losses in the condenser are usually small. These include the loss of pressure and the cooling of condensate below the saturation temperature.

12.4 COMPARISON OF RANKINE AND CARNOT CYCLES

Although the Carnot cycle has the maximum possible efficiency for the given limits of temperature, it is not suitable in steam power plants. Figure 12.9 shows the Rankine and Carnot cycles on the *T–s* diagram. The reversible adiabatic expansion in the turbine, the constant temperature heat rejection in the condenser, and the reversible adiabatic compression in the pump, are similar characteristic features of both the Rankine

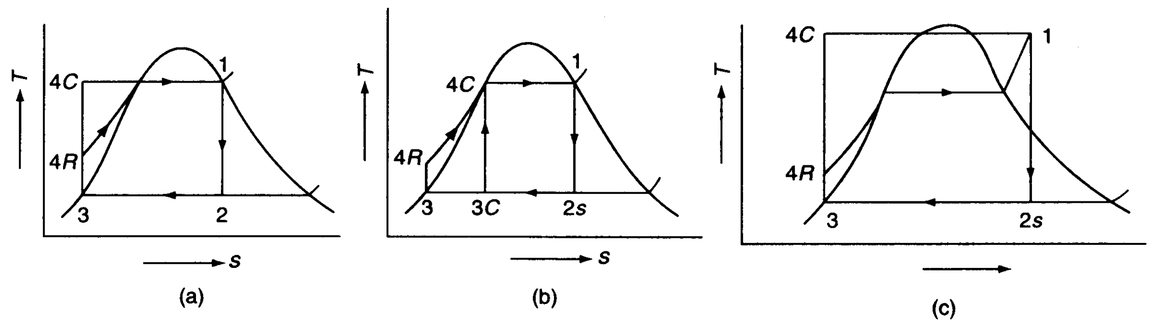


Fig. 12.9 *Comparison of Carnot and Rankine cycles*

and Carnot cycles. But whereas the heat addition process in the Rankine cycle is reversible and at constant pressure, in the Carnot cycle it is reversible and isothermal. In Figs 12.9(a) and 12.9(c), Q_2 is the same in both the cycles, but since Q_1 is more, η_{Carnot} is greater than η_{Rankine}. The two Carnot cycles in Fig. 12.9(a) and 12.9(b) have the same thermal efficiency. Therefore, in Fig. 12.9(b) also, $\eta_{\text{Carnot}} > \eta_{\text{Rankine}}$. But the Carnot cycle cannot be realized in practice because the pump work [in all the three cycles (a), (b), and (c)] is very large. Whereas in (a) and (c) it is impossible to add heat at infinite pressures and at constant temperature from state 4*c* to state 1, in (b), it is difficult to control the quality at 3*c*, so that insentropic compression leads to saturated liquid state.

12.5 MEAN TEMPERATURE OF HEAT ADDITION

In the Rankine cycle, heat is added reversibly at a constant pressure, but at infinite temperatures. If T_{m1} is the mean temperature of heat addition, as shown in Fig. 12.10, so that the area under 4*s* and 1 is equal to the area under 5 and 6, then heat added

$$Q_1 = h_1 - h_{4s} = T_{m1}(s_1 - s_{4s})$$

$\therefore$ T_{m1} = Mean temperature of heat addition

$$= \frac{h_1 - h_{4s}}{s_1 - s_{4s}}$$

Since Q_2 = Heat rejected $= h_{2s} - h_3$

$$= T_2(s_1 - s_{4s})$$

$$\eta_{\text{Rankine}} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2(s_1 - s_{4s})}{T_{m1}(s_1 - s_{4s})}$$

$$\therefore \quad \eta_{\text{Rankine}} = 1 - \frac{T_2}{T_{m1}} \qquad (12.13)$$

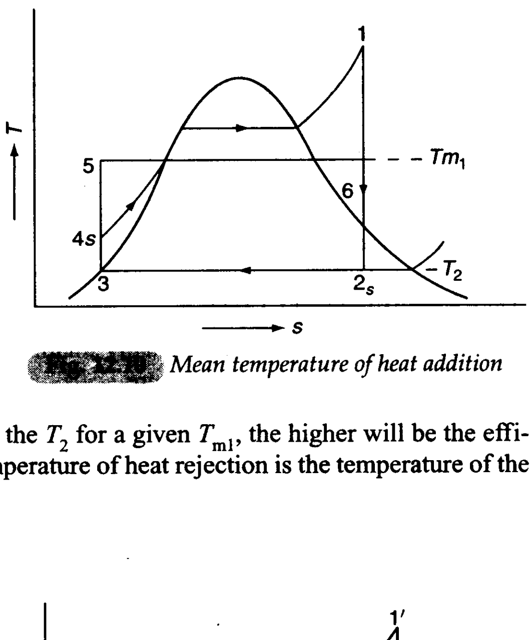


Fig. 12.10 *Mean temperature of heat addition*

where T_2 is the temperature of heat rejection. The lower is the T_2 for a given T_{m1}, the higher will be the efficiency of the Rankine cycle. But the lowest practicable temperature of heat rejection is the temperature of the surroundings (T_0). This being fixed,

$$\eta_{\text{Rankine}} = f(T_{m1}) \text{ only} \qquad (12.14)$$

The higher the mean temperature of heat addition, the higher will be the cycle efficiency.

The effect of increasing the initial temperature at constant pressure on cycle efficiency is shown in Fig. 12.11. When the initial state changes from 1 to 1′, T_{m1} between 1 and 1′ is higher than T_{m1} between 4s and 1. So an increase in the superheat at constant pressure increases the mean temperature of heat addition and hence the cycle efficiency.

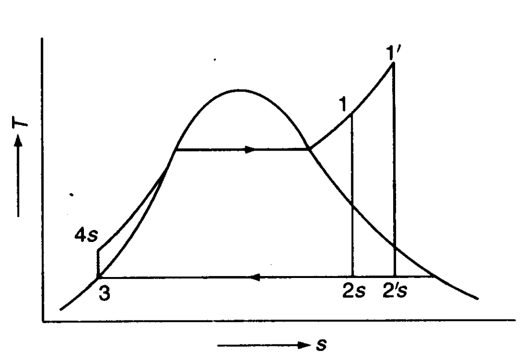


Fig. 12.11 *Effect of superheat on mean temperature of heat addition*

The maximum temperature of steam that can be used is *fixed from metallurgical considerations* (i.e. the materials used for the manufacture of the components which are subjected to high-pressure, high temperature steam like the superheaters, valves, pipelines, inlet stages of turbine, etc.). When the maximum temperature is fixed, as the operating steam pressure at which heat is added in the boiler increases from p_1 to p_2 (Fig. 12.12), the mean temperature of heat addition increases, since T_{m1} between 7*s* and 5 is higher than that between 4s and 1. But when the turbine inlet pressure increases from p_1 to p_2, the ideal expansion line shifts to the left and the moisture content at the turbine exhaust increases (because $x_{6s} < x_{2s}$). If the moisture content of steam in the later stages of the turbine is higher, the entrained water particles along with the vapour coming out from the nozzles with high velocity strike the blades and erode their surfaces, as a result of which the longevity of the blades decreases. From a consideration of the erosion of blades in the later stages of the turbine, the maximum moisture content at the turbine exhaust is not allowed to exceed 15%, or the quality to fall below 85%. It is desirable that most of the turbine expansion should take place in the single phase or vapour region.

Therefore, with the maximum steam temperature at the turbine inlet, the minimum temperature of heat rejection, and the minimum quality of steam at the turbine exhaust being fixed, the maximum steam pressure at the turbine inlet also gets fixed (Fig. 12.13). The vertical line drawn from 2*s*, fixed by T_2 and x_{2s}, intersects the T_{max} line, fixed by material, at 1, which gives the maximum steam pressure at the turbine inlet. The irreversibility in the expansion process has, however, not been considered.

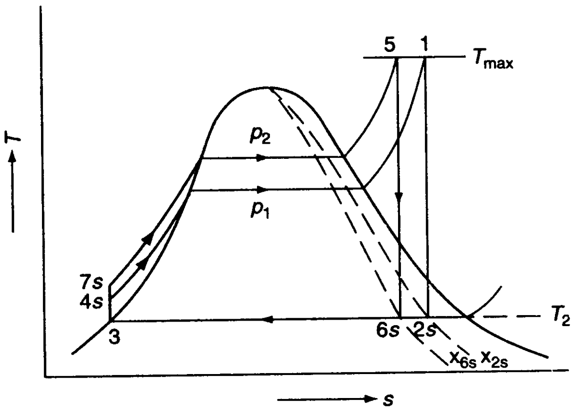


Fig. 12.12 *Effect of increase of pressure on Rankine cycle*

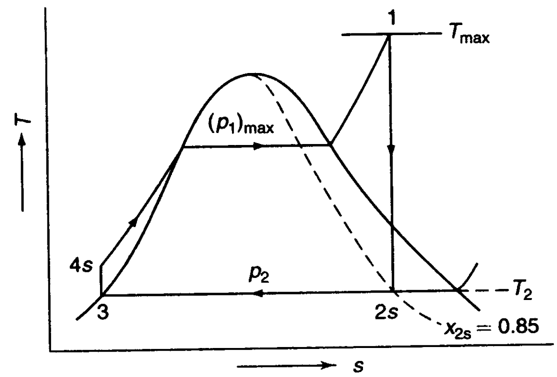


Fig. 12.13 *Fixing of exhaust quality, maximum temperature and maximum pressure in Ranking cycle*

12.6 REHEAT CYCLE

If a steam pressure higher than $(p_1)_{max}$ (Fig. 12.13) is used, in order to limit the quality to 0.85, at the turbine exhaust, reheat has to be adopted. In that case all the steam after partial expansion in the turbine is brought back to the boiler, reheated by combustion gases and then fed back to the turbine for further expansion. The flow, *T–s*, and *h–s* diagrams for the ideal Rankine cycle with reheat are shown in Fig. 12.14. In the reheat cycle the expansion of steam from the initial state 1 to the condenser pressure is carried out in two or more steps, depending upon the number of reheats used. In the first step, steam expands in the high pressure (H.P.) turbine from the initial state to approximately the saturated vapour line (process 1–2*s* in Fig. 12.14). The steam is then resuperheated (or reheated) at constant pressure in the boiler (process 2*s*–3) and the remaining expansion (process 3–4*s*) is carried out in the low pressure (L.P.) turbine. In the case of use of two reheats, steam is resuperheated twice at two different constant pressures. To protect the reheat tubes, steam is not allowed to expand deep into the two-phase region before it is taken for reheating, because in that case the moisture particles in steam while evaporating

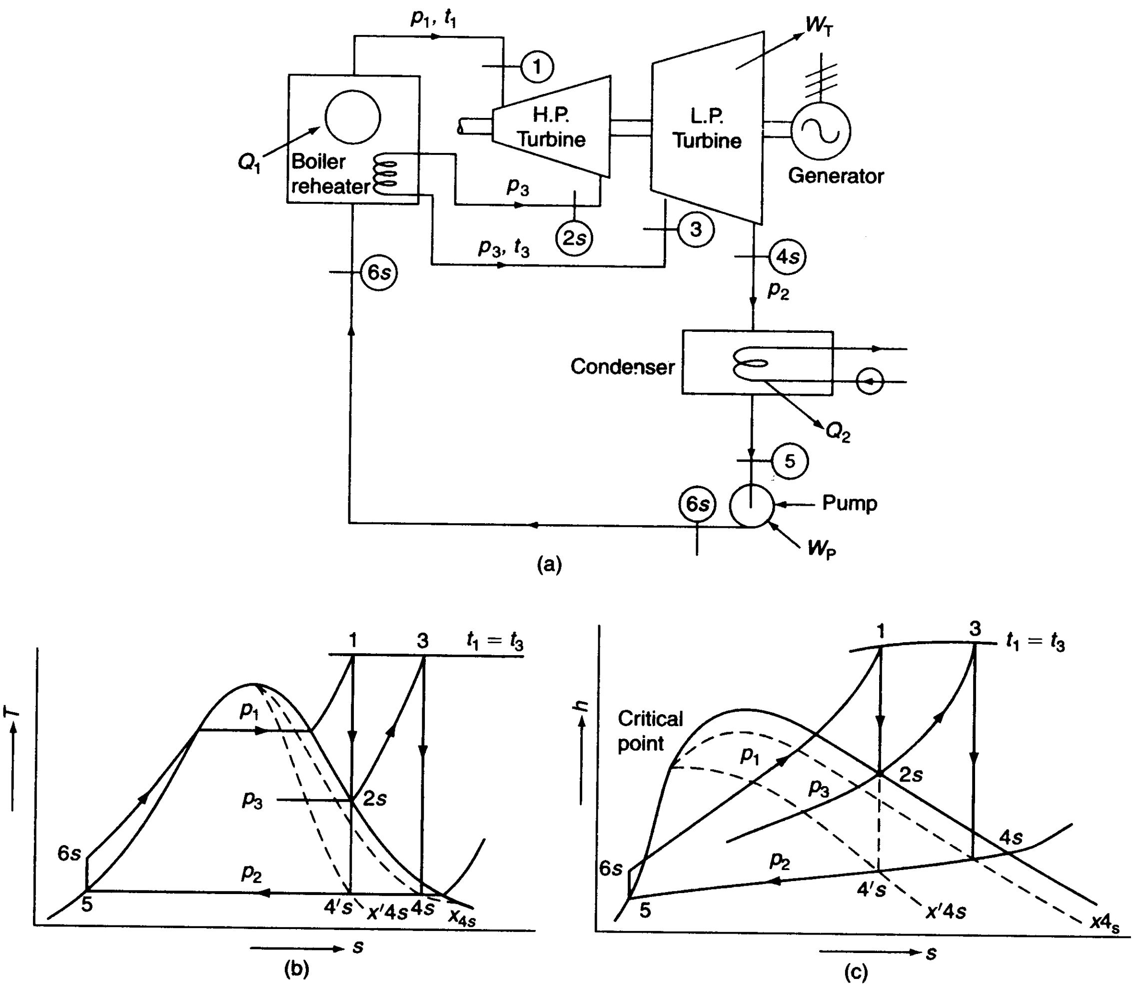


Fig. 12.14 *Reheat cycle*

would leave behind solid deposits in the form of scale which is difficult to remove. Also, a low reheat pressure may bring down T_{m1} and hence, cycle efficiency. Again, a high reheat pressure increases the moisture content at turbine exhaust. Thus, the reheat pressure is optimized. The optimum reheat pressure for most of the modern power plants is about 0.2 to 0.25 of the initial steam pressure. For the cycle in Fig. 12.14, for 1 kg of steam

$$Q_1 = h_1 - h_{6s} + h_3 - h_{2s}$$

$$Q_2 = h_{4s} - h_5$$

$$W_T = h_1 - h_{2s} + h_3 - h_{4s}$$

$$W_P = h_{6s} - h_5$$

$$\eta = \frac{W_T - W_P}{Q_1} = \frac{(h_1 - h_{2s} + h_3 - h_{4s}) - (h_{6s} - h_5)}{h_1 - h_{6s} + h_3 - h_{2s}} \tag{12.15}$$

$$\text{Steam rate} = \frac{3600}{(h_1 - h_{2s} + h_3 - h_{4s}) - (h_{6s} - h_5)} \text{ kg/kWh} \tag{12.16}$$

where enthalpy is in kJ/kg.

Since higher pressures are used in a reheat cycle, pump work may be appreciable.

Had the high pressure p_1 been used without reheat, the ideal Rankine cycle would have been $1 - 4's - 5 - 6s$. With the use of reheat, the area $2s - 3 - 4s - 4's$ has been added to the basic cycle. It is obvious that net work output of the plant increases with reheat, because $(h_3 - h_{4s})$ is greater than $(h_{2s} - h_{4's})$, and hence the steam rate decreases. Whether the cycle efficiency improves with reheat depends upon whether the mean temperature of heat addition in process $2s - 3$ is higher than the mean temperature of heat addition in process $6s - 1$. In practice, the use of reheat only gives a small increase in cycle efficiency, but it increases the net work output by making possible the use of higher pressures, keeping the quality of steam at turbine exhaust within a permissible limit. The quality improves from $x_{4's}$ to x_{4s} by the use of reheat.

By increasing the number of reheats, still higher steam pressures could be used, but the mechanical stresses increase at a higher proportion than the increase in pressure, because of the prevailing high temperature. The cost and fabrication difficulties will also increase. In that way, the maximum steam pressure gets fixed, and more than two reheats have not yet been used so far.

In Fig. 12.14, only ideal processes have been considered. The irreversibilities in the expansion and compression processes have been considered in the example given later.

12.7 IDEAL REGENERATIVE CYCLE

In order to increase the mean temperature of heat addition (T_{m1}), attention was so far confined to increasing the amount of heat supplied at high temperatures, such as increasing superheat, using higher pressure and temperature of steam, and using reheat. The mean temperature of heat addition can also be increased by decreasing the amount of heat added at low temperatures. In a saturated steam Rankine cycle (Fig. 12.15), a considerable part of the total heat supplied is in the liquid phase when heating up water from 4 to 4′, at a temperature lower than T_1, the maximum temperature of the cycle. For maximum efficiency, all heat should be supplied at T_1, and feedwater should enter the boiler at state 4′. This may be accomplished in what is known as an ideal regenerative cycle, the flow diagram of which is shown in Fig. 12.16 and the corresponding T–s diagram in Fig. 12.17.

The unique feature of the ideal regenerative cycle is that the condensate, after leaving the pump circulates around the turbine casing, counterflow to the direction of vapour flow in the turbine (Fig. 12.16).

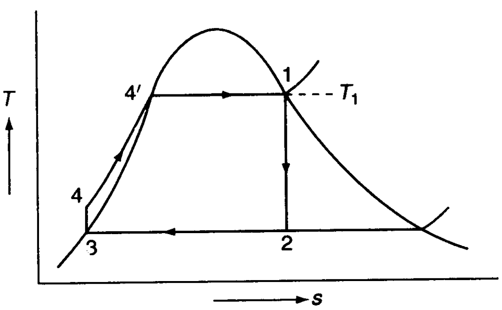


Fig. 12.15 Simple Rankine cycle

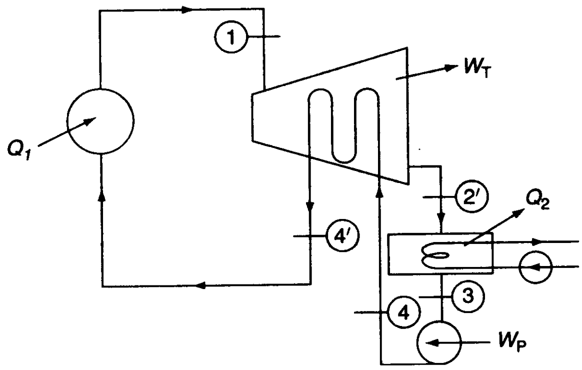


Fig. 12.16 Ideal regenerative cycle-basic scheme

Thus it is possible to transfer heat from the vapour as it flows through the turbine to the liquid flowing around the turbine. Let us assume that this is a reversible heat transfer, i.e. at each point the temperature of the vapour is only infinitesimally higher than the temperature of the liquid. The process 1–2′ (Fig. 12.17) thus represents reversible expansion of steam in the turbine with reversible heat rejection. For any small step in the process of heating the water,

$$\Delta T \text{ (water)} = -\ \Delta T \text{ (steam)}$$

and $$\Delta s \text{ (water)} = -\ \Delta s \text{ (steam)}$$

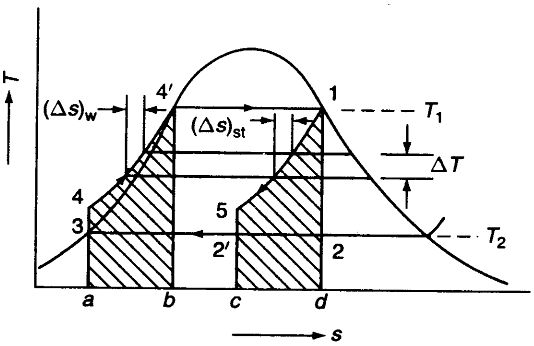


Fig. 12.17 Ideal regenerative cycle on T–s plot

Then the slopes of lines 1–2′ and 4′–3 (Fig. 12.17) will be identical at every temperature and the lines will be identical in contour. Areas 4–4′–b–a–4 and 2′–1–d–c–2′ are not only equal but congruous. Therefore, all the heat added from an external source (Q_1) is at the constant temperature T_1, and all the heat rejected (Q_2) is at the constant temperature T_2, both being reversible.

Then

$$Q_1 = h_1 - h_{4'} = T_1(s_1 - s_{4'})$$

$$Q_2 = h_{2'} - h_3 = T_2(s_{2'} - s_3)$$

Since

$$s_{4'} - s_3 = s_1 - s_{2'}$$

or

$$s_1 - s_{4'} = s_{2'} - s_3$$

$$\therefore \quad \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

The efficiency of the ideal regenerative cycle is thus equal to the Carnot cycle efficiency.

Writing the steady flow energy equation for the turbine

$$h_1 - W_T - h_{2'} + h_4 - h_{4'} = 0$$

or

$$W_T = (h_1 - h_{2'}) - (h_{4'} - h_4) \tag{12.17}$$

The pump work remains the same as in the Rankine cycle, i.e.

$$W_P = h_4 - h_3$$

The net work output of the ideal regenerative cycle is thus less, and hence its steam rate will be more, although it is more efficient, when compared with the Rankine cycle. However, the cycle is not practicable for the following reasons:

(a) Reversible heat transfer cannot be obtained in finite time.
(b) Heat exchanger in the turbine is mechanically impracticable.
(c) The moisture content of the steam in the turbine will be high.

12.8 REGENERATIVE CYCLE

In the practical regenerative cycle, the feedwater enters the boiler at a temperature between 4 and 4′ (Fig. 12.17), and it is heated by steam extracted from intermediate stages of the turbine. The flow diagram of

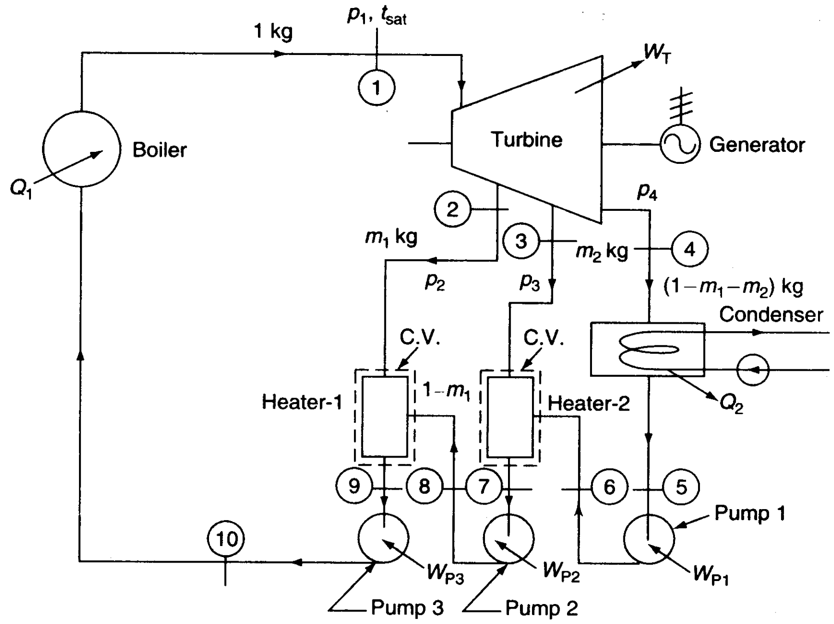


Fig. 12.18 *Regenerative cycle flow diagram with two feedwater heaters*

the regenerative cycle with saturated steam at the inlet to the turbine, and the corresponding T–s diagram are shown in Figs 12.18 and 12.19 respectively. For every kg of steam entering the turbine, let m_1 kg steam be extracted from an intermediate stage of the turbine where the pressure is p_2, and it is used to heat up feedwater $[(1 - m_1)$ kg at state 8] by mixing in heater 1. The remaining $(1 - m_1)$ kg of steam then expands in the turbine from pressure p_2 (state 2) to pressure p_3 (state 3) when m_2 kg of steam is extracted for heating feedwater in heater 2. So $(1 - m_1 - m_2)$ kg of steam then expands in the remaining stages of the turbine to pressure p_4, gets condensed into water in the condenser, and then pumped to heater 2, where it mixes with m_2 kg of steam extracted at pressure p_3. Then $(1 - m_1)$ kg of water is pumped to heater 1 where it mixes with m_1 kg

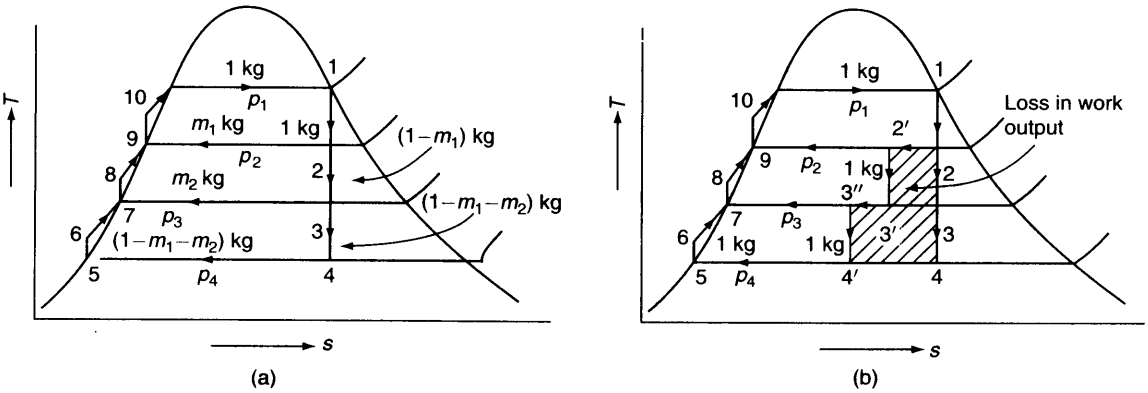


Fig. 12.19 *(a) Regenerative cycle on T–s plot with decreasing mass of fluid*
(b) Regenerative cycle on T–s plot for unit mass of fluid

of steam extracted at pressure p_2. The resulting 1 kg of steam is then pumped to the boiler where heat from an external source is supplied. Heaters 1 and 2 thus operate at pressures p_2 and p_3 respectively. The amounts of steam m_1 and m_2 extracted from the turbine are such that at the exit from each of the heaters, the state is saturated liquid at the respective pressures. The heat and work transfer quantities of the cycle are

$$W_T = 1\,(h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1 - m_2)(h_3 - h_4) \text{ kJ/kg} \quad (12.18)$$

$$W_P = W_{P1} + W_{P2} + W_{P3}$$

$$= (1 - m_1 - m_2)(h_6 - h_5) + (1 - m_1)(h_8 - h_7) + 1\,(h_{10} - h_9) \text{ kJ/kg} \quad (12.19)$$

$$Q_1 = 1\,(h_1 - h_{10}) \text{ kJ/kg} \quad (12.20)$$

$$Q_2 = (1 - m_1 - m_2)(h_4 - h_5) \text{ kJ/kg} \quad (12.21)$$

Cycle efficiency,

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{W_T - W_P}{Q_1}$$

$$\text{Steam rate} = \frac{3600}{W_T - W_P} \text{ kg/kW h}$$

In the Rankine cycle operating at the given pressures, p_1 and p_4, the heat addition would have been from state 6 to state 1. By using two stages of regenerative feedwater heating, feedwater enters the boiler at state 10, instead of state 6, and heat addition is, therefore, from state 10 to state 1. Therefore

$$(T_{m_1})_{\text{with regeneration}} = \frac{h_1 - h_{10}}{s_1 - s_{10}} \quad (12.22)$$

and

$$(T_{m_1})_{\text{without regeneration}} = \frac{h_1 - h_6}{s_1 - s_6} \quad (12.23)$$

Since

$$(T_{m_1})_{\text{with regeneration}} > (T_{m_1})_{\text{without regeneration}}$$

the efficiency of the regenerative cycle will be higher than that of the Rankine cycle.

The energy balance for heater 2 gives

$$m_1 h_2 + (1 - m_1)\, h_8 = 1 h_9$$

$$m_1 = \frac{h_9 - h_8}{h_2 - h_8} \quad (12.24)$$

The energy balance for heater 1 gives

$$m_2 h_3 + (1 - m_1 - m_2)\, h_6 = (1 - m_1)\, h_7$$

or

$$m_2 = (1 - m_1)\frac{h_7 - h_6}{h_3 - h_6} \quad (12.25)$$

From Eqs (12.24) and (12.25), m_1 and m_2 can be evaluated. Equations (12.24) and (12.25) can also be written alternatively as

$$(1 - m_1)(h_9 - h_8) = m_1 (h_2 - h_9)$$

$$(1 - m_1 - m_2)(h_7 - h_6) = m_2 (h_3 - h_7)$$

Energy gain of feedwater = Energy given off by vapour in condensation. Heaters have been assumed to be adequately insulated, and there is no heat gain from, or heat loss to, the surroundings.

Path 1–2–3–4 in Fig. 12.19 represents the states of a decreasing mass of fluid. For 1 kg of steam, the states would be represented by the path 1–2′–3′–4′. From Equation (12.18),

$$W_T = (h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1 - m_2)(h_3 - h_4)$$
$$= (h_1 - h_2) + (h_{2'} - h_{3'}) + (h_{3'} - h_{4'}) \quad (12.26)$$

where
$$(1 - m_1)(h_2 - h_3) = 1\,(h_{2'} - h_{3'}) \quad (12.27)$$
$$(1 - m_1 - m_2)(h_3 - h_4) = 1\,(h_{3''} - h_{4'}) \quad (12.28)$$

The cycle 1–2–2′–3′–3″–4′–5–6–7–8–9–10–1 represents 1 kg of working fluid. The heat released by steam condensing from 2 to 2′ is utilized in heating up the water from 8 to 9.

∴
$$1(h_2 - h_{2'}) = 1(h_9 - h_8) \quad (12.29)$$

Similarly
$$1(h_{3'} - h_{3''}) = 1(h_7 - h_6) \quad (12.30)$$

From Eqs (12.26), (12.29) and (12.30)
$$W_T = (h_1 - h_{4'}) - (h_2 - h_{2'}) - (h_{3'} - h_{3''})$$
$$= (h_1 - h_{4'}) - (h_9 - h_8) - (h_7 - h_6) \quad (12.31)$$

The similarity of Eqs (12.17) and (12.31) can be noticed. It is seen that the stepped cycle 1–2′–3′–4′–5–6–7–8–9–10 approximates the ideal regenerative cycle in Fig. 12.17, and that a greater number of stages would give a closer approximation (Fig. 12.20). Thus the heating of feedwater by steam 'bled' from the turbine, known as regeneration, *carnotizes* the Rankine cycle.

The heat rejected Q_2 in the cycle decreases from $(h_4 - h_5)$ to $(h_{4'} - h_5)$. There is also loss in work output by the amount (Area under 2–2′ + Area under 3′–3″ – Area under 4–4′), as shown by the hatched area in Fig. 12.19(b). So the steam rate increases by regeneration, i.e. more steam has to circulate per hour to produce unit shaft output.

The enthalpy-entropy diagram of a regenerative cycle is shown in Fig. 12.21.

12.9 REHEAT-REGENERATIVE CYCLE

The reheating of steam is adopted when the vaporization pressure is high. The effect of reheat alone on the thermal efficiency of the cycle is very small. Regeneration or the heating up of feedwater by steam extracted from the turbine has a marked effect on cycle efficiency. A modern steam power plant is equipped with both. Figures 12.22 and 12.23 give the flow and T–s diagrams of a steam plant with reheat and three stages of feedwater heating. Here

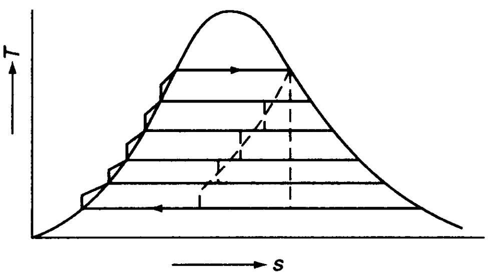


Fig. 12.20 Regenerative cycle with many stages of feedwater heating

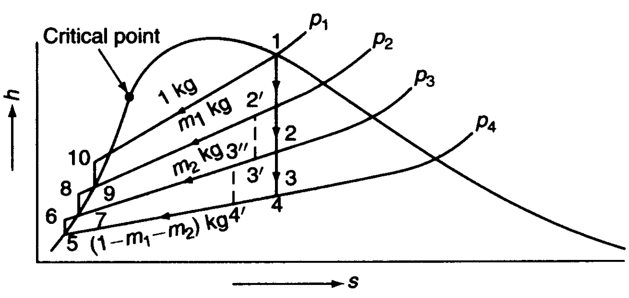


Fig. 12.21 Regenerative cycle on h–s diagram

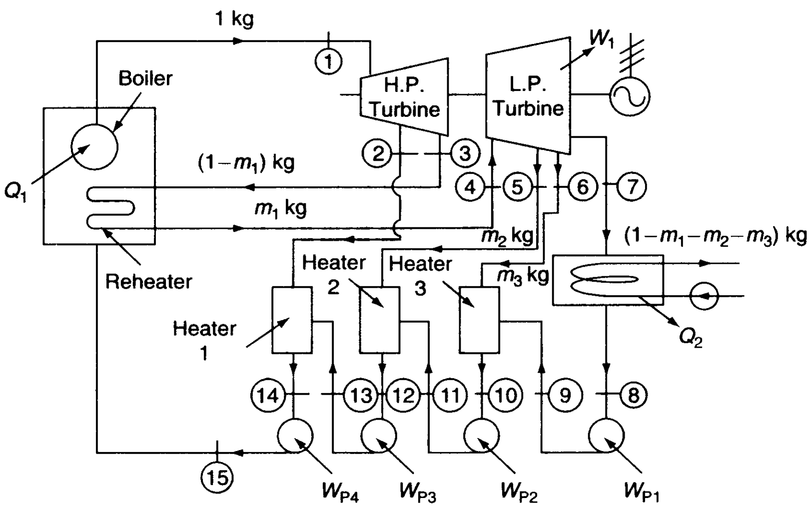


Fig. 12.22 *Reheat-regenerative cycle flow diagram*

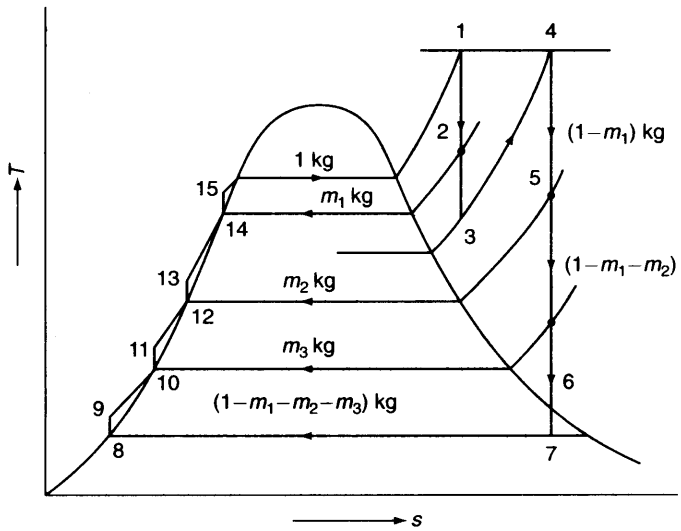


Fig. 12.23 *T–s diagram of reheat-regenerative cycle*

$$W_T = (h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1)(h_4 - h_5)$$
$$+ (1 - m_1 - m_2)(h_5 - h_6) + (1 - m_1 - m_2 - m_3)(h_6 - h_7) \text{ kJ/kg}$$

$$W_P = (1 - m_1 - m_2 - m_3)(h_9 - h_8) + (1 - m_1 - m_2)(h_{11} - h_{10})$$
$$+ (1 - m_1)(h_{13} - h_{12}) + 1(h_{15} - h_{14}) \text{ kJ/kg}$$

$$Q_1 = (h_1 - h_{15}) + (1 - m_1)(h_4 - h_3) \text{ kJ/kg}$$

and

$$Q_2 = (1 - m_1 - m_2 - m_3)(h_7 - h_8) \text{ kJ/kg}$$

The energy balance of heaters 1, 2, and 3 give

$$m_1 h_2 + (1 - m_1)h_{13} = 1 \times h_{14}$$
$$m_2 h_5 + (1 - m_1 - m_2)h_{11} = (1 - m_1)h_{12}$$
$$m_3 h_6 + (1 - m_1 - m_2 - m_3)h_9 = (1 - m_1 - m_2)h_{10}$$

from which m_1, m_2, and m_3 can be evaluated.

12.10 FEEDWATER HEATERS

Feedwater heaters are of two types, viz., open heaters and closed heaters. In an open or contact-type heater, the extracted or bled steam is allowed to mix with feedwater, and both leave the heater at a common temperature, as shown in Figs 12.18 and 12.22. In a closed heater, the fluids are kept separate, and not allowed to mix together (Fig. 12.24). The feedwater flows through the tubes in the heater and the extracted steam condenses on the outside of the tubes in the shell. The heat released by condensation is transferred to the feedwater through the walls of the tubes. The condensate (saturated water at the steam extraction pressure), sometimes called the heater-drip, then passes through a trap into the next lower pressure heater. This, to some extent, reduces the steam required by that heater. The trap passes only liquid and no vapour. The drip from the lowest pressure heater could similarly be trapped to the condenser, but this would be throwing away energy to the condenser cooling water. To avoid this waste, a drip pump feeds the drip directly into the feedwater stream.

Figure 12.25 shows the *T–s* plot corresponding to the flow diagram in Fig. 12.24. The temperature of the feedwater (at '*l*' or '*o*') leaving a particular heater is always less than the saturation temperature at the steam extraction pressure (at *e* or *g*), the difference being known as the *terminal temperature difference* of the heater.

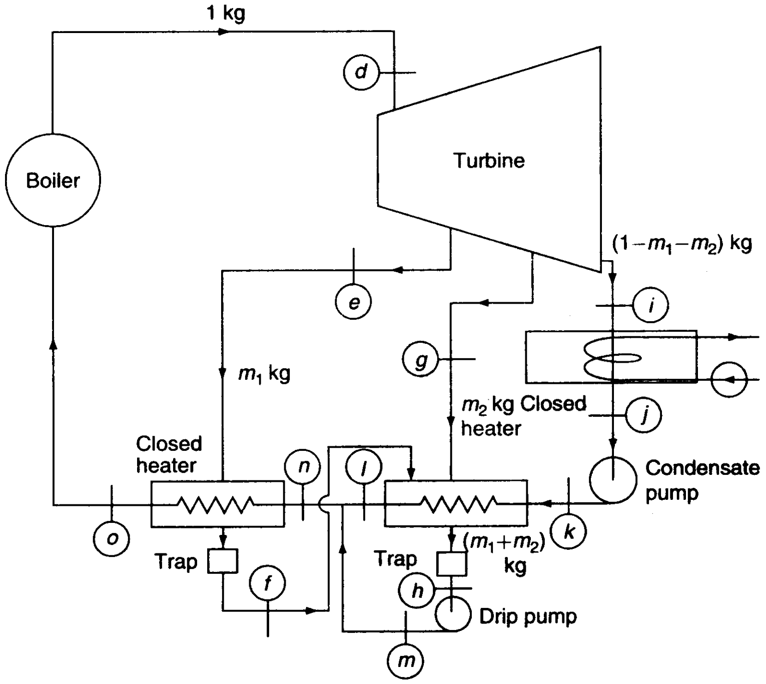
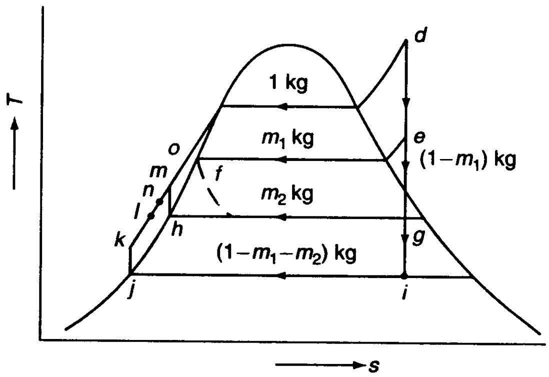


Fig. 12.24 *Regenerative cycle flow diagram with closed feedwater heaters*

The advantages of the open heater are simplicity, lower cost, and high heat transfer capacity. The disadvantage is the necessity of a pump at each heater to handle the large feedwater stream.

A closed heater system requires only a single pump for the main feedwater stream regardless of the number of heaters. The drip pump, if used, is relatively small. Closed heaters are costly and may not give as high a feedwater temperature as do open heaters. In most steam power plants, closed heaters are favoured, but at least one open heater is used, primarily for the purpose of feedwater deaeration. The open heater in such a system is called the *deaerator*.



T–s diagram of regenerative cycle with closed feedwater heaters

The higher the number of heaters used, the higher will be the cycle efficiency. If n heaters are used, the greatest gain in efficiency occurs when the overall temperature rise is about $n/(n + 1)$ times the difference between the condenser and boiler saturation temperatures. (See *Analysis of Engineering Cycles* by R.W. Haywood, Pergamon Press, 1973).

If $(\Delta f)_0 = t_{\text{boiler sat}} - t_{\text{cond}}$ and $(\Delta t)_{\text{fw}}$ = temperature rise of feedwaters, it is seen that.

$$n = 0 \quad (\Delta t)_{fw} = 0$$

$$n = 1, \quad (\Delta t)_{fw} = \frac{1}{2}(\Delta t)_0$$

Gain $= \frac{1}{6}(\Delta t)_0$

$$n = 2, \quad (\Delta t)_{fw} = \frac{2}{3}(\Delta t)_0$$

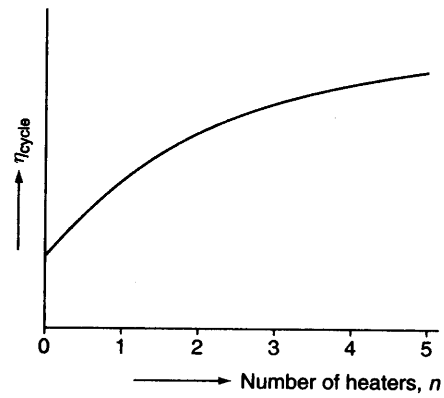
Gain $= \frac{1}{12}(\Delta t)_0$

$$n = 3, \quad (\Delta t)_{fw} = \frac{3}{4}(\Delta t)_0$$

Gain $= \frac{1}{20}(\Delta t)_0$

$$n = 4, \quad (\Delta t)_{fw} = \frac{4}{2}(\Delta t)_0$$

Since the cycle efficiency is proportional to $(\Delta t)_{\text{fw}}$ the efficiency gain follows *the law of diminishing return* with the increase in the number of heaters. The greatest increment in efficiency occurs by the use of the first heater (Fig. 12.26). The increments for each additional heater thereafter successively diminish. The number of heaters is fixed up by the energy balance of the whole plant when it is found that the cost of adding another does not justify the saving in Q_1 or the marginal increase in cycle efficiency. An increase in feedwater temperature may, in some cases, cause a reduction in boiler efficiency. So, the number of heaters gets optimized. Five points of extraction are often used in practice. Some cycles use as many as nine.



Effect of the use of number of heaters on cycle efficiency

12.11 EXERGY ANALYSIS OF VAPOUR POWER CYCLES

Let the heating for steam generation in the boiler unit be provided by a stream of hot gases produced by burning of a fuel (Fig. 12.27). The distribution of input energy is shown in the Sankey diagram 12.27 (b) which indicates that only about 30% of the input energy to the simple ideal plant is converted to shaft work and about 60% is lost to the condenser. The exergy analysis, however, gives a different distribution as discussed below.

Assuming that the hot gases are at atmospheric pressure, the exergy input is

$$a_{f_1} = w_g c_{p_g}\left[T_i - T_0 - T_0 \ln\frac{T_i}{T_0}\right]$$

$$= w_g c_{p_g} T_0\left[\frac{T_i}{T_0} - 1 - \ln\frac{T_i}{T_0}\right]$$

Similarly, the exergy loss rate with the exhaust stream is:

$$a_{f_2} = w_g c_{p_g} T_0\left[\frac{T_e}{T_0} - 1 - \ln\frac{T_e}{T_0}\right]$$

Net exergy input rate in the steam generation process:

$$a_{i_1} = a_{f_1} - a_{f_2}$$

The exergy utilization rate in the steam generation is:

$$a_{f_u} = w_s[(h_1 - h_4) - T_0(s_1 - s_4)]$$

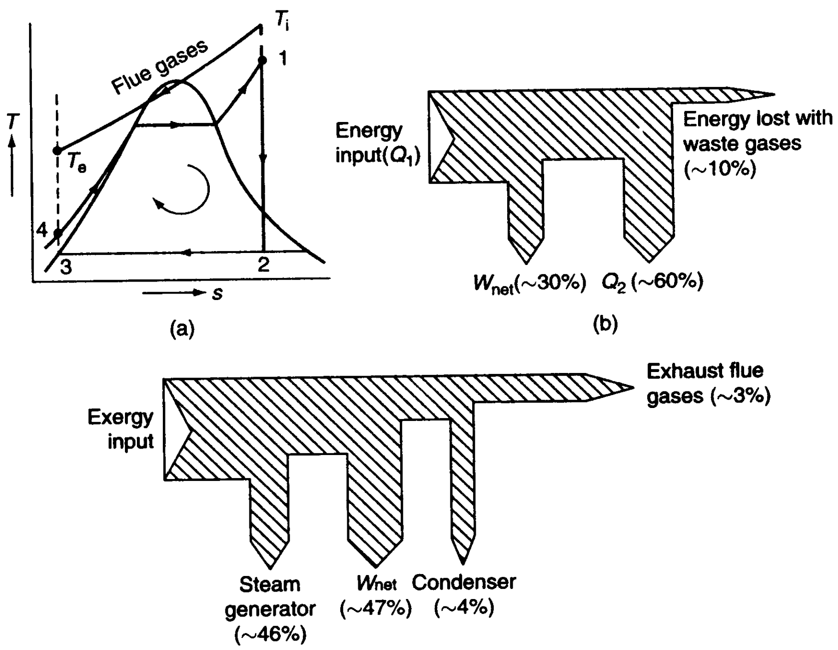


Fig. 12.27 (a) T–s diagram, (b) Sankey diagram, (c) Grassman diagram

Rate of exergy loss in the steam generator:

$$I = a_{i_1} - a_{f_u}$$

The useful mechanical power output:

$$= W_{net} = w_s[(h_1 - h_2) - (h_4 - h_3)]$$

Exergy flow rate of the wet steam to the condenser:

$$a_{f_c} = w_s[(h_2 - h_3) - T_0(s_2 - s_3)]$$

Second law efficiency,

$$\eta_{II} = \frac{W_{net}}{a_{f_1} - a_{f_2}}$$

Exergy flow or Grassman diagram is shown in Fig. 12.27 (c). The energy disposition diagram (b) shows that the major energy loss (~60%) takes place in the condenser. This energy rejection, however, occurs at a temperature close to the ambient temperature, and, therefore, corresponds to a very low exergy value (~4%). The major exergy destruction due to irreversibilities takes place in the steam generation. To improve the performance of the steam plant the finite source temperatures must be closer to the working fluid temperatures to reduce thermal irreversibility.

12.12 CHARACTERISTICS OF AN IDEAL WORKING FLUID IN VAPOUR POWER CYCLES

There are certain drawbacks with steam as the working substance in a power cycle. The maximum temperature that can be used in steam cycles consistent with the best available material is about 600°C, while the critical temperature of steam is 375°C, which necessitates large superheating and permits the addition of only an infinitesimal amount of heat at the highest temperature.

High moisture content is involved in going to higher steam pressures in order to obtain higher mean temperature of heat addition (T_{m1}). The use of reheat is thus necessitated. Since reheater tubes are costly, the use of more than two reheats is hardly recommended. Also, as pressure increases, the metal stresses increase, and the thicknesses of the walls of boiler drums, tubes, pipe lines, etc. increase not in proportion to pressure increase, but much faster, because of the prevalence of high temperature.

It may be noted that high T_{m1} is only desired for high cycle efficiency. High pressures are only forced by the characteristics (weak) of steam.

If the lower limit is now considered, it is seen that at the heat rejection temperature of 40°C, the saturation pressure of steam is 0.075 bar, which is considerably lower than atmospheric pressure. The temperature of heat rejection can be still lowered by using some refrigerant as a coolant in the condenser. The corresponding vacuum will be still higher, and to maintain such low vacuum in the condenser is a big problem.

It is the low temperature of heat rejection that is of real interest. The necessity of a vacuum is a disagreeable characteristic of steam.

The saturated vapour line in the T–s diagram of steam is sufficiently inclined, so that when steam is expanded to lower pressures (for higher turbine output as well as cycle efficiency), it involves more moisture content, which is not desired from the consideration of the erosion of turbine blades in later stages.

The desirable characteristics of the working fluid in a vapour power cycle to obtain best thermal efficiency are given below:

(a) The fluid should have a high critical temperature so that the saturation pressure at the maximum permissible temperature (metallurgical limit) is relatively low. It should have a large enthalpy of evaporation at that pressure.

(b) The saturation pressure at the temperature of heat rejection should be above atmospheric pressure so as to avoid the necessity of maintaining vacuum in the condenser.

(c) The specific heat of liquid should be small so that little heat transfer is required to raise the liquid to the boiling point.

(d) The saturated vapour line of the *T–s* diagram should be steep, very close to the turbine expansion process so that excessive moisture does not appear during expansion.

(e) The freezing point of the fluid should be below room temperature, so that it does not get solidified while flowing through the pipelines.

(f) The fluid should be chemically stable and should not contaminate the materials of construction at any temperature.

(g) The fluid should be nontoxic, noncorrosive, not excessively viscous, and low in cost.

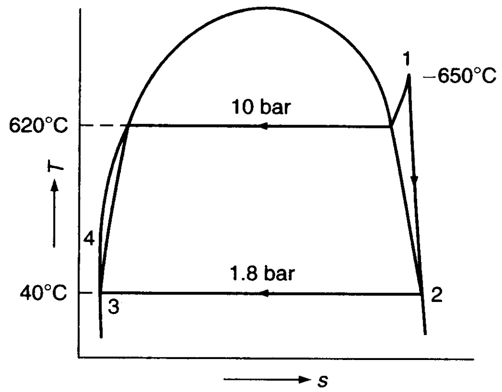


Fig. 12.28 *T–s diagram of an ideal working fluid for a vapour power cycle*

The characteristics of such an ideal fluid are approximated in the *T-s* diagram as shown in Fig. 12.28. Some superheat is desired to reduce piping losses and improve turbine efficiency. The thermal efficiency of the cycle is very close to the Carnot efficiency.

12.13 BINARY VAPOUR CYCLES

No single fluid can meet all the requirements as mentioned above. Although in the overall evaluation, water is better than any other working fluid, however, in the high temperature range, there are a few better fluids, and notable among them are (a) diphenyl ether, $(C_6H_5)_2O$, (b) aluminium bromide, Al_2Br_6, and (c) mercury and other liquid metals like sodium or potassium. From among these, only mercury has actually been used in practice. Diphenyl ether could be considered, but it has not yet been used because, like most organic substances, it decomposes gradually at high temperatures. Aluminium bromide is a possibility and yet to be considered.

When $p = 12$ bar, the saturation temperature for water, aluminium bromide, and mercury are 187°C, 482.5°C, and 560°C respectively. Mercury is thus a better fluid in the high temperature range, because at high temperature, its vaporization pressure is relatively low. Its critical pressure and temperature are 1080 bar and 1460°C respectively.

But in the low temperature range, mercury is unsuitable, because its saturation pressure becomes exceedingly low and it would be impractical to maintain such a high vacuum in the condenser. At 30°C, the saturation pressure of mercury is only 2.7×10^{-4} cm Hg. Its specific volume at such a low pressure is very large, and it would be difficult to accommodate such a large volume flow.

For this reason, mercury vapour leaving the mercury turbine is condensed at a higher temperature, and the heat released during the condensation of mercury is utilized in evaporating water to form steam to operate on a conventional turbine.

Thus in the binary (or two-fluid) cycle, two cycles with different working fluids are coupled in series, the heat rejected by one being utilized in the other.

The flow diagram of mercury-steam binary cycle and the corresponding *T–s* diagram are given in Figs 12.29 and 12.30 respectively. The mercury cycle, *a–b–c–d*, is a simple Rankine type of cycle using saturated vapour. Heat is supplied to the mercury in process *d–a*. The mercury expands in a turbine (process *a–b*) and is then condensed in process *b–c*. The feed pump process, *c–d*, completes the cycle.

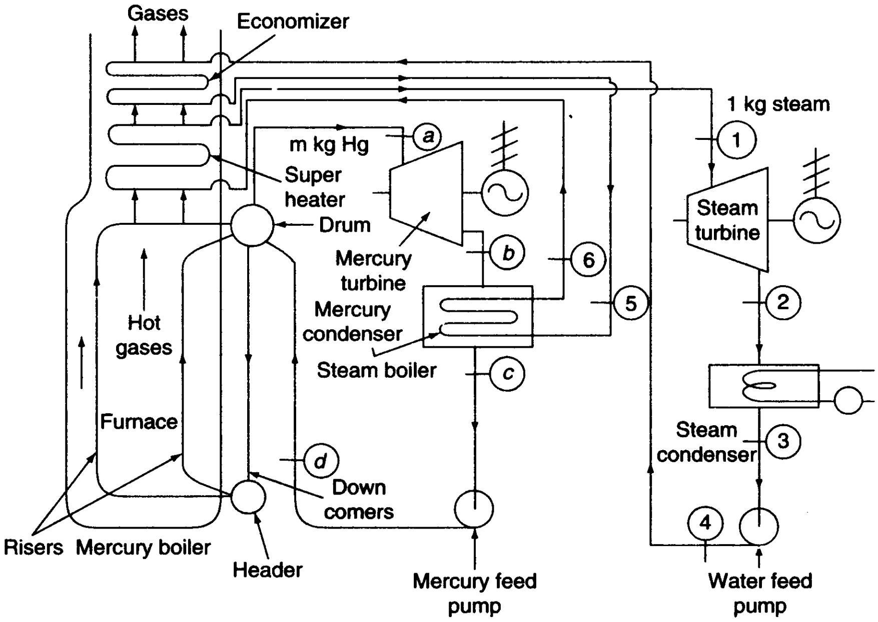


Fig. 12.29 *Mercury-steam plant flow diagram*

The heat rejected by mercury during condensation is transferred to boil water and form saturated vapour (process 5–6). The saturated vapour is heated from the external source (furnace) in the superheater (process 6–1). Superheated steam expands in the turbine (process 1–2) and is then condensed (process 2–3). The feedwater (condensate) is then pumped (process 3–4), heated till it is saturated liquid in the economizer (process 4–5) before going to the mercury condenser-steam boiler, where the latent heat is absorbed. In an actual plant the steam cycle is always a regenerative cycle, but for the sake of simplicity, this complication has been omitted.

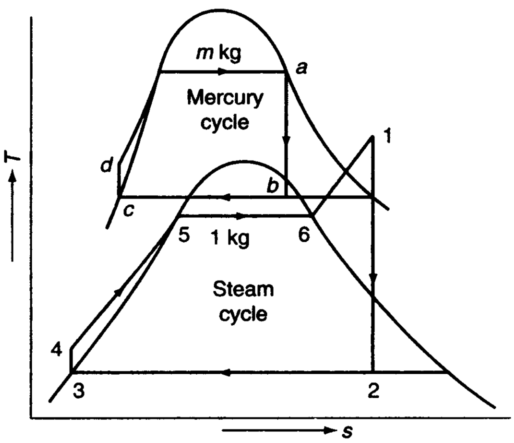


Fig. 12.30 *Mercury-steam binary cycle*

Let m represent the flow rate of mercury in the mercury cycle per kg of steam circulating in the steam cycle. Then for 1 kg of steam

$$Q_1 = m(h_a - h_d) + (h_1 - h_6) + (h_5 - h_4)$$

$$Q_2 = h_2 - h_3$$

$$W_T = m(h_a - h_b) + (h_1 - h_2)$$

$$W_P = m(h_d - h_c) + (h_4 - h_3)$$

$$\eta_{cycle} = \frac{Q_1 - Q_2}{Q_1} = \frac{W_T - W_P}{Q_1}$$

and steam rate $= \dfrac{3600}{W_T - W_P}$ kg/kWh

The energy balance of the mercury condenser-steam boiler gives

$$m(h_b - h_c) = h_6 - h_5$$

$$\therefore \quad m = \frac{h_6 - h_5}{h_b - h_c} \text{ kg Hg/kg } H_2O$$

To vaporize one kg of water, seven to eight kg of mercury must condense.

The addition of the mercury cycle to the steam cycle results in a marked increase in the mean temperature of heat addition to the plant as a whole and consequently the efficiency is increased. The maximum pressure is relatively low.

It may be interesting to note that the concept of the binary vapour cycle evolved from the need of improving the efficiency of the reciprocating steam engine. When steam expands up to, say, atmospheric temperature, the resultant volume flow rate of steam becomes too large for the steam engine cylinder to accommodate. So most of the early steam engines are found to be non-condensing. The binary cycle with steam in the high temperature and ammonia or sulphur dioxide in the low temperature range, was first suggested by Professor Josse of Germany in the middle of the nineteenth century. Steam exhausted from the engine at a relatively higher pressure and temperature was used to evaporate ammonia or sulphur dioxide which operated on another cycle. But with the progress in steam turbinc design, such a cycle was found to be of not much utility, since modern turbines can cope efficiently with a large volume flow of steam.

The mercury-steam cycle has been in actual commercial use for more than three decades. One such plant is the Schiller Station in the USA. But it has never attained wide acceptance because there has always been the possibility of improving steam cycles by increasing pressure and temperature, and by using reheat and regeneration. Over and above, mercury is expensive, limited in supply, and highly toxic.

The mercury-steam cycle represents the two-fluid cycles. The mercury cycle is called the *topping cycle* and the steam cycle is called the *bottoming cycle*. If a sulphur dioxide cycle is added to it in the low temperature range, so that the heat released during the condensation of steam is utilized in forming sulphur dioxide vapour which expands in another turbine, then the mercury-steam-sulphur dioxide cycle is a three-fluid or tertiary cycle. Similarly, other liquid metals, apart from mercury, like sodium or potassium, may be considered for a working fluid in the topping cycle. Apart from SO_2 other refrigerants (ammonia, freons, etc.) may be considered as working fluids for the bottoming cycle.

Since the possibilities of improving steam cycles are diminishing, and the incentives to reduce fuel cost are very much increasing, coupled cycles, like the mercury-steam cycle, may receive more favourable consideration in the near future.

12.14 THERMODYNAMICS OF COUPLED CYCLES

If two cycles are coupled in series where heat lost by one is absorbed by the other (Fig. 12.31), as in the mercury-steam binary cycle, let η_1 and η_2 be the efficiencies of the topping and bottom cycles respectively, and η be the overall efficiency of the combined cycle.

$$\eta_1 = 1 - \frac{Q_2}{Q_1} \quad \text{and} \quad \eta_2 = 1 - \frac{Q_3}{Q_2}$$

or

$$Q_2 = Q_1(1 - \eta_1) \quad \text{and} \quad Q_3 = Q_2(1 - \eta_2)$$

Now

$$\eta = 1 - \frac{Q_3}{Q_1} = 1 - \frac{Q_2(1-\eta_2)}{Q_1}$$

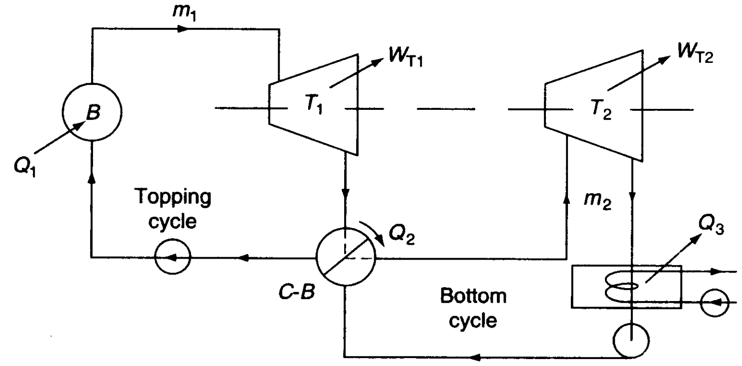


Fig. 12.31 Two vapour cycles coupled in series

$$= 1 - \frac{Q_1(1-\eta_1)(1-\eta_2)}{Q_1}$$

$$= 1 - (1 - \eta_1)(1 - \eta_2)$$

If there are n cycles coupled in series, the overall efficiency would be given by

$$\eta = 1 - \prod_{i=1}^{n}(1-\eta_i)$$

i.e.
$$\eta = 1 - (1 - \eta_1)(1 - \eta_2)(1 - \eta_3) \ldots (1 - \eta_n)$$

or
$$1 - \eta = (1 - \eta_1)(1 - \eta_2)(1 - \eta_3) \ldots (1 - \eta_n)$$

$\therefore$ Total loss = Product of losses in all the cycles.

For two cycles coupled in series

$$\eta = 1 - (1 - \eta_1)(1 - \eta_2)$$
$$= 1 - (1 - \eta_1 - \eta_2 + \eta_1\eta_2)$$
$$= \eta_1 + \eta_2 - \eta_1\eta_2$$

or
$$\eta = \eta_1 + \eta_2 - \eta_1\eta_2$$

This shows that the overall efficiency of two cycles coupled in series equals the sum of the individual efficiencies minus their product.

By combining two cycles in series, even if individual efficiencies are low, it is possible to have a fairly high combined efficiency, which cannot be attained by a single cycle.

For example, if $\eta_1 = 0.50$ and $\eta_2 = 0.40$

$$\eta = 0.5 + 0.4 - 0.5 \times 0.4 = 0.70$$

It is almost impossible to achieve such a high efficiency in a single cycle.

12.15 PROCESS HEAT AND BY-PRODUCT POWER

There are several industries, such as paper mills, textile mills, chemical factories, dying plants, rubber manufacturing plants, sugar factories, etc., where saturated steam at the desired temperature is required for heating, drying, etc. For constant temperature heating (or drying), steam is a very good medium, since isothermal condition can be maintained by allowing saturated steam to condense at that temperature and utilizing the latent heat released for heating purposes. Apart from the process heat, the factory also needs power to drive various machines, for lighting, and for other purposes.

Formerly it was the practice to generate steam for power purposes at a moderate pressure and to generate separately saturated steam for process work at a pressure which gave the desired heating temperature. Having two separate units for process heat and power is wasteful, for of the total heat supplied to the steam for power purposes, a greater part will normally be carried away by the cooling water in the condenser.

By modifying the initial steam pressure and exhaust pressure, it is possible to generate the required power and make available for process work the required quantity of exhaust steam at the desired temperature. In Fig. 12.32, the exhaust steam from the turbine is utilized for process heating, the process heater replacing the condenser of the ordinary Rankine cycle. The pressure at exhaust from the turbine is the saturation pressure corresponding to the temperature desired in the process heater. Such a turbine is called *a back pressure turbine*. A plant producing both power and process heat is sometimes known as a *cogeneration plant*. When the process steam is the basic need, and the power is produced incidentally as a by-product, the cycle is sometimes called a by-product power cycle. Figure 12.33 shows the *T–s* plot of such a cycle. If W_T is the turbine output in kW, Q_H the process heat required in kJ/h, and w is the steam flow rate in kg/h

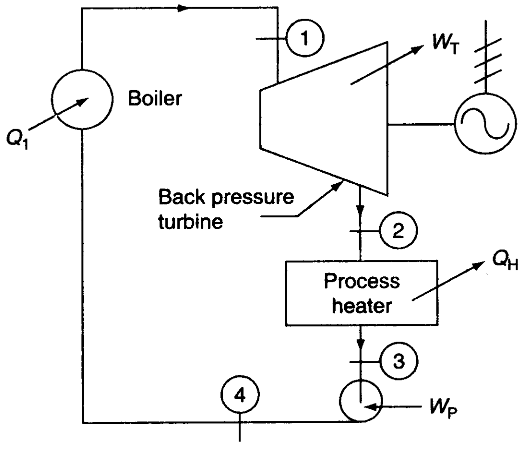


Fig. 12.32 Back pressure turbine

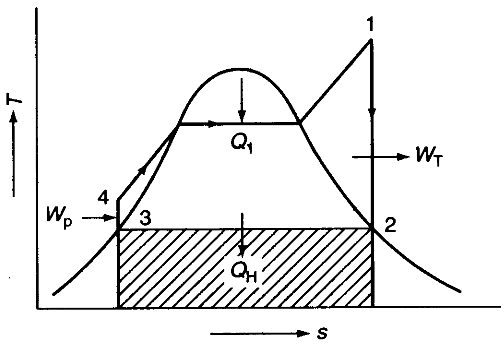


Fig. 12.33 By-product power cycle

$$W_T \times 3600 = w(h_1 - h_2)$$

and

$$w(h_2 - h_3) = Q_H$$

$\therefore$

$$W_T \times 3600 = \frac{Q_H}{h_2 - h_3}(h_1 - h_2)$$

or

$$Q_H = \frac{W_T \times 3600 \times (h_2 - h_3)}{h_1 - h_2} \text{ kJ/h}$$

Of the total energy input Q_1 (as heat) to the by-product cycle, W_T part of it only is converted into shaft work (or electricity). The remaining energy $(Q_1 - W_T)$, which would otherwise have been a waste, as in the Rankine cycle (by the Second Law), is utilized as process heat.

Fraction of energy (Q_1) utilized in the form of work (W_T), and process heat (Q_H) in a by-product power cycle

$$= \frac{W_T + Q_H}{Q_1}$$

Condenser loss, which is the biggest loss in a steam plant, is here zero, and the fraction of energy utilized is very high.

In many cases the power available from the back pressure turbine through which the whole of the heating steam flows is appreciably less than that required in the factory. This may be due to relatively high back pressure, or small heating requirement, or both. *Pass-out turbines* are employed in these cases, where a certain quantity of steam is continuously extracted for heating purposes at the desired temperature and pressure. (Figs 12.34 and 12.35).

$$Q_1 = w(h_1 - h_8) \text{ kJ/h}$$
$$Q_2 = (w - w_1)(h_3 - h_4) \text{ kJ/h}$$
$$Q_H = w_1(h_2 - h_6) \text{ kJ/h}$$
$$W_T = w(h_1 - h_2) + (w - w_1)(h_2 - h_3) \text{ kJ/h}$$
$$W_P = (w - w_1)(h_5 - h_4) + w_1(h_7 - h_6) \text{ kJ/h}$$
$$w_1 h_7 + (w - w_1)h_5 = w \times h_8$$

where w is the boiler capacity (kg/h) and w_1 is the steam flow rate required (kg/h) at the desired temperature for process heating.

12.16 EFFICIENCIES IN STEAM POWER PLANT

For the steady flow operation of a turbine, neglecting changes in K.E. and P.E. (Figs 12.36 and 12.37).

Maximum or ideal work output per unit mass of steam

$$(W_T)_{max} = (W_T)_{ideal} = h_1 - h_{2s}$$
$$= \text{Reversible and adiabatic enthalpy drop in turbine}$$

This work is, however, not obtainable, since no real process is reversible. The expansion process is accompanied by irreversibilities. The actual final state 2 can be defined, since the temperature, pressure, and quality can be found by actual measurement. The actual path 1–2 is not known and its nature is immaterial, since the work output is here being expressed in terms of the change of a property, enthalpy. Accordingly, the work done by the turbine in irreversible adiabatic expansion from 1 to 2 is

$$(W_T)_{actual} = h_1 - h_2$$

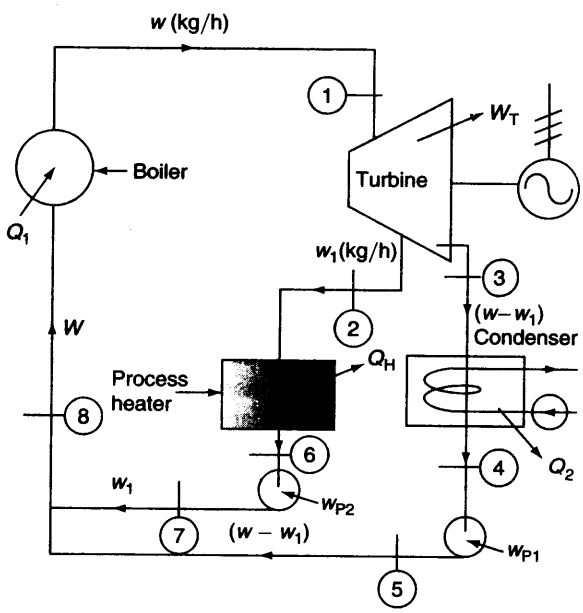


Fig. 12.34 *Pass-out turbine*

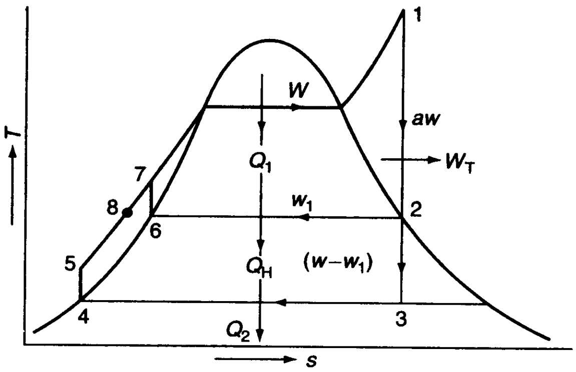


Fig. 12.35 *T–s diagram of power and process heat plant*

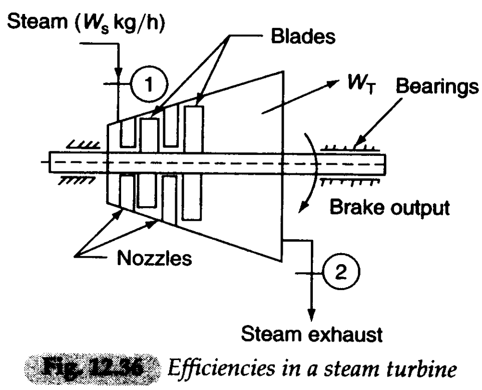


Fig. 12.36 *Efficiencies in a steam turbine*

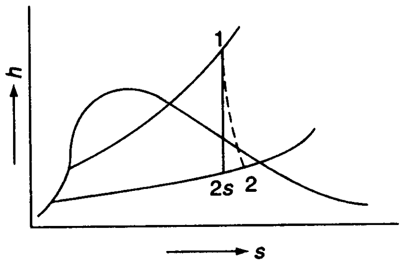
This work is known as *internal work*, since only the irreversibilities within the flow passages of turbine are affecting the state of steam at the turbine exhaust.

$\therefore$ Internal output = Ideal output – Friction and other losses within the turbine casing

If w_s is the steam flow rate in kg/h

Internal output = $w_s(h_1 - h_2)$ kJ/h

Ideal output = $w_s(h_1 - h_{2s})$ kJ/h



Internal efficiency of a steam turbine

The *internal efficiency* of turbine is defined as

$$\eta_{\text{internal}} = \frac{\text{Internal output}}{\text{Ideal output}} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

Work output available at the shaft is less than the internal output because of the external losses in the bearings, etc.

$\therefore$ Brake output or shaft output

= Internal output – External losses

= Ideal output – Internal and External losses

= (kW × 3600 kJ/h)

The *brake efficiency* of turbine is defined as

$$\eta_{\text{brake}} = \frac{\text{Brake output}}{\text{Ideal output}}$$

$$= \frac{\text{kW} \times 3600}{w_s(h_1 - h_{2s})}$$

The *mechanical efficiency* of turbine is defined as

$$\eta_{\text{mech}} = \frac{\text{Brake output}}{\text{Internal output}}$$

$$= \frac{\text{kW} \times 3600}{w_s(h_1 - h_2)}$$

$$\therefore \quad \eta_{\text{brake}} = \eta_{\text{internal}} \times \eta_{\text{mech}}$$

While the internal efficiency takes into consideration the internal losses, and the mechanical efficiency considers only the external losses, the brake efficiency takes into account both the internal and external losses (with respect to turbine casing).

The generator (or *alternator*) efficiency is defined as

$$\eta_{\text{generator}} = \frac{\text{Output at generator terminals}}{\text{Brake output of turbine}}$$

The *boiler efficiency* is defined as

$$\eta_{\text{boiler}} = \frac{\text{Energy utilized}}{\text{Energy supplied}} = \frac{w_s(h_1 - h_4)}{w_f \times \text{C.V.}}$$

where w_f is the fuel burning rate in the boiler (kg/h) and C.V. is the calorific value of the fuel (kJ/kg), i.e. the heat energy released by the complete combustion of unit mass of fuel.

The power plant is an *energy converter* from fuel to electricity (Fig. 12.38), and the *overall efficiency* of the plant is defined as

$$\eta_{overall} = \eta_{plant} = \frac{kW \times 3600}{w_f \times C.V.}$$

This may be expressed as

$$\eta_{overall} = \frac{kW \times 3600}{w_f \times C.V.} = \frac{w(h_1 - h_4)}{w_f \times C.V.} \times \frac{w(h_1 - h_2)}{w_s(h_1 - h_4)}$$

$$\times \frac{\text{Brake output}}{w_s(h_1 - h_2)} \times \frac{kW \times 3600}{\text{Brake output}}$$

or $\eta_{overall} = \eta_{boiler} \times \eta_{cycle} \times \eta_{turbine\,(mech)} \times \eta_{generator}$

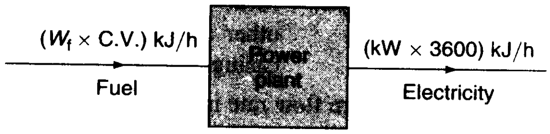


Fig. 12.38 *Power plant—an energy converter from fuel to electricity*

where pump work has been neglected in the expression for cycle efficiency.

Solved Examples

Example 12.1

Determine the work required to compress steam isentropically from 1 *bar to* 10 *bar, assuming that at the initial state the steam exists as* (*a*) *saturated liquid and* (*b*) *saturated vapour. Neglect changes in kinetic and potential energies. What conclusion do you derive from this example?*

Solution The compression processes are shown in Fig. Ex. 12.1

(a) Steam is a saturated liquid initially, and its specific volume is:

$$v_1 = (v_f)_{1\,bar} = 0.001043 \text{ m}^3\text{/kg}$$

Since liquid is incompressible, v_1 remains constant.

$$W_{rev} = -\int_1^2 v dp = v_1(p_1 - p_2)$$

$$= 0.001043\,(1 - 10) \times 10^2 = -\,0.9387 \text{ kJ/kg.}$$

(b) Steam is a saturated vapour initially and remains a vapour during the entire compression process. Since the specific volume of a gas changes considerably during a compression process, we need to know how v varies with p to perform the integration $-\int v\,dp$. This relation is not readily available. But for an isentropic process, it is easily obtained from the property relation

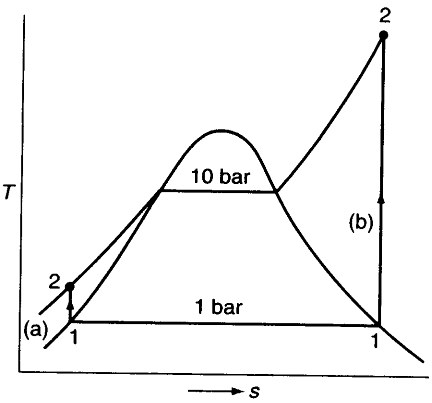


Fig. Ex. 12.1 *Compression of steam isentropically*

$$T\,ds = dh - v dp = 0$$

or

$$v\,dp = dh$$

$$W_{rev} = -\int_1^2 v\,dp = -\int_1^2 dh = h_1 - h_2$$

From steam tables,

$$h_1 = (h_g)_{1\text{bar}} = 2675.5 \text{ kJ/kg}$$

$$s_1 = (s_g)_{1\text{bar}} = 7.3594 \text{ kJ/kg K} = s_2$$

For $p = 10$ bar $= 1$ MPa and $s = 7.3594$ kJ/kg K, by interpolation

$$h_2 = 3195.5 \text{ kJ/kg}$$

$$W_{\text{rev}} = 2675.5 - 3195.5 = -520 \text{ kJ/kg}$$

It is thus observed that compressing steam in vapour form would require over 500 times more work than compressing it in liquid form for the same pressure rise.

Example 12.2

Steam at 20 bar, 360°C is expanded in a steam turbine to 0.08 bar. It then enters a condenser, where it is condensed to saturated liquid water. The pump feeds back the water into the boiler. (a) Assuming ideal processes, find per kg of steam the net work and the cycle efficiency. (b) If the turbine and the pump have each 80% efficiency, find the percentage reduction in the net work and cycle efficiency.

Solution The property values at different state points (Fig. Ex. 12.2) found from the steam tables are given below.

$$h_1 = 3159.3 \text{ kJ/kg} \qquad s_1 = 6.9917 \text{ kJ/kg K}$$

$$h_3 = h_{fp2} = 173.88 \text{ kJ/kg} \qquad s_3 = s_{fp2} = 0.5926 \text{ kJ/kg K}$$

$$h_{fgp2} = 2403.1 \text{ kJ/kg} \qquad s_{gp2} = 8.2287 \text{ kJ/kg K}$$

$$v_{fp2} = 0.001008 \text{ m}^3\text{/kg} \qquad \therefore s_{fgp2} = 7.6361 \text{ kJ/kg K}$$

Now $\quad s_1 = s_{2s} = 6.9917 = s_{fp2} + x_{2s}\, s_{fgp2} = 0.5926 + x_2 \cdot 7.6361$

$$\therefore \quad x_{2s} = \frac{6.3991}{7.6361} = 0.838$$

$$\therefore \quad h_{2s} = h_{fp2} + x_{2s} h_{fgp2} = 173.88 + 0.838 \times 2403.1$$

$$= 2187.68 \text{ kJ/kg}$$

(a) $W_P = h_{4s} - h_3 = v_{fp2}(p_1 - p_2)$

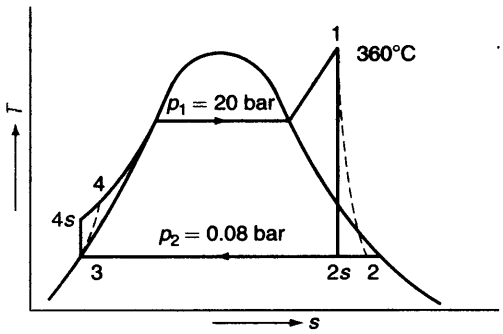
$$= 0.001008 \frac{\text{m}^3}{\text{kg}} \times 19.92 \times 100 \frac{\text{kN}}{\text{m}^2}$$

$$= 2.008 \text{ kJ/kg}$$

$$h_{4s} = 175.89 \text{ kJ/kg}$$

$$W_T = h_1 - h_{2s}$$

$$= 3159.3 - 2187.68 = 971.62 \text{ kJ/kg}$$



$\therefore W_{net} = W_T - W_P = 969.61$ kJ/kg

$Q_1 = h_1 - h_{4s} = 3159.3 - 175.89$

$= 2983.41$ kJ/kg *Ans.*

$$\therefore \eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{969.61}{2983.41} = 0.325, \text{ or } 32.5\%$$ *Ans.*

(b) If $\eta_P = 80\%$, and $\eta_T = 80\%$

$$W_P = \frac{2.008}{0.8} = 2.51 \text{ kJ/kg}$$

$$W_T = 0.8 \times 971.62 = 777.3 \text{ kJ/kg}$$

$$\therefore \quad W_{net} = W_T - W_P = 774.8 \text{ kJ/kg}$$

$\therefore$ % Reduction in work output

$$= \frac{969.61 - 774.8}{969.61} \times 100 = 20.1\%$$ *Ans.*

$$h_{4s} = 173.88 + 2.51 = 176.39 \text{ kJ/kg}$$

$$\therefore \quad Q_1 = 3159.3 - 176.39 = 2982.91 \text{ kJ/kg}$$

$$\therefore \quad \eta_{cycle} = \frac{774.8}{2982.91} = 0.2597, \text{ or } 25.97\%$$

$\therefore$ % Reduction in cycle efficiency

$$= \frac{0.325 - 0.2597}{0.325} \times 100 = 20.1\%$$ *Ans.*

Example 12.3

A cyclic steam power plant is to be designed for a steam temperature at turbine inlet of 360°C and an exhaust pressure of 0.08 bar. After isentropic expansion of steam in the turbine, the moisture content at the turbine exhaust is not to exceed 15%. Determine the greatest allowable steam pressure at the turbine inlet, and calculate the Rankine cycle efficiency for these steam conditions. Estimate also the mean temperature of heat addition.

Solution As state 2*s* (Fig. Ex. 12.3), the quality and pressure are known.

$$\therefore \quad s_{2s} = s_f + x_{2s}\, s_{fg} = 0.5926 + 0.85\,(8.2287 - 0.5926)$$

$$= 7.0833 \text{ kJ/kg K}$$

Since $s_1 = s_{2s}$

$$\therefore \quad s_1 = 7.0833 \text{ kJ/kg K}$$

At state 1, the temperature and entropy are thus known. At 360°C, s_g = 5.0526 kJ/kg K, which is less than s_1. So from the table of superheated steam, at t_1 = 360°C and s_1 = 7.0833 kJ/kg K, the pressure is found to be 16.832 bar (by interpolation).

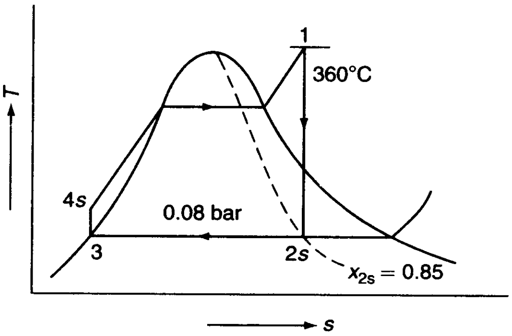


Fig. Ex. 12.3

∴ The greatest allowable steam pressure is

$$p_1 = 16.832 \text{ bar}$$
$$h_1 = 3165.54 \text{ kJ/kg}$$
$$h_{2s} = 173.88 + 0.85 \times 2403.1$$
$$= 2216.52 \text{ kJ/kg}$$
$$h_3 = 173.88 \text{ kJ/kg}$$
$$h_{4s} - h_3 = 0.001 \times (16.83 - 0.08) \times 100 = 1.675 \text{ kJ/kg}$$
$$\therefore \quad h_{4s} = 175.56 \text{ kJ/kg}$$
$$Q_1 = h_1 - h_{4s} = 3165.54 - 175.56$$
$$= 2990 \text{ kJ/kg}$$
$$W_T = h_1 - h_{2s} = 3165.54 - 2216.52 = 949 \text{ kJ/kg}$$
$$W_P = 1.675 \text{ kJ/kg}$$
$$\eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{247.32}{2990} = 0.3168 \text{ or } 31.68\%$$ *Ans.*

Mean temperature of heat addition

$$T_{m1} = \frac{h_1 - h_{4s}}{s_1 - s_{4s}} = \frac{2990}{7.0833 - 0.5926}$$
$$= 460.66 \text{ K} = 187.51°\text{C}.$$

Example 12.4

A steam power station uses the following cycle:

Steam at boiler outlet—150 bar, 550°C

Reheat at 40 bar to 550°C

Condenser at 0.1 bar.

Using the Mollier chart and assuming ideal processes, find the (a) quality at turbine exhaust, (b) cycle efficiency, and (c) steam rate.

Solution The property values at different states (Fig. Ex. 12.4) are read from the Mollier chart.

$$h_1 = 3465, h_{2s} = 3065, h_3 = 3565,$$
$$h_{4s} = 2300 \text{ kJ/kg } x_{4s} = 0.88, h_5(\text{steam table}) = 191.83 \text{ kJ/kg}$$

Quality at turbine exhaust = 0.88 *Ans.* (a)

$$W_P = v\Delta p = 10^{-3} \times 150 \times 10^2 = 15 \text{ kJ/kg}$$
$$\therefore \quad h_{6s} = 206.83 \text{ kJ/kg}$$
$$Q_1 = (h_1 - h_{6s}) + (h_3 - h_{2s})$$
$$= (3465 - 206.83) + (3565 - 3065) = 3758.17 \text{ kJ/kg}$$

$$W_T = (h_1 - h_{2s}) + (h_3 - h_{4s})$$
$$= (3465 - 3065) + (3565 - 2300) = 1665 \text{ kJ/kg}$$

$\therefore \quad W_{net} = W_T - W_P = 1665 - 15 = 1650$ kJ/kg

$\therefore \quad \eta_{cycle} = \dfrac{W_{net}}{Q_1} = \dfrac{1650}{3758.17} = 0.4390$, or 43.9% *Ans.* (b)

Steam rate $= \dfrac{3600}{1650} = 2.18$ kg/kW h *Ans.* (c)

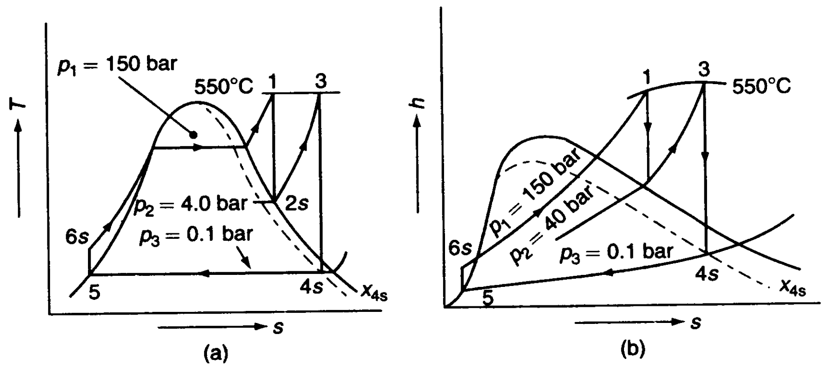


Fig. Ex. 12.4

Example 12.5

In a single-heater regenerative cycle the steam enters the turbine at 30 *bar,* 400°*C and the exhaust pressure is* 0.10 *bar. The feed water heater is a direct-contact type which operates at* 5 *bar. Find* (*a*) *the efficiency and the steam rate of the cycle and* (*b*) *the increase in mean temperature of heat addition, efficiency and steam rate, as compared to the Rankine cycle* (*without regeneration*). *Neglect pump work.*

Solution Figure Ex. 12.5 gives the flow, T–s, and h–s diagrams. From the steam tables, the property values at various states have been obtained.

$$h_1 = 3230.9 \text{ kJ/kg}$$
$$s_1 = 6.9212 \text{ kJ/kg K} = s_2 = s_3$$
$$s_g \text{ at 5 bar} = 6.8213 \text{ kJ/kg K}$$

Since $s_2 > s_g$, the state 2 must lie in the superheated region. From the table for superheated steam $t_2 = 172$°C, $h_2 = 2796$ kJ/kg

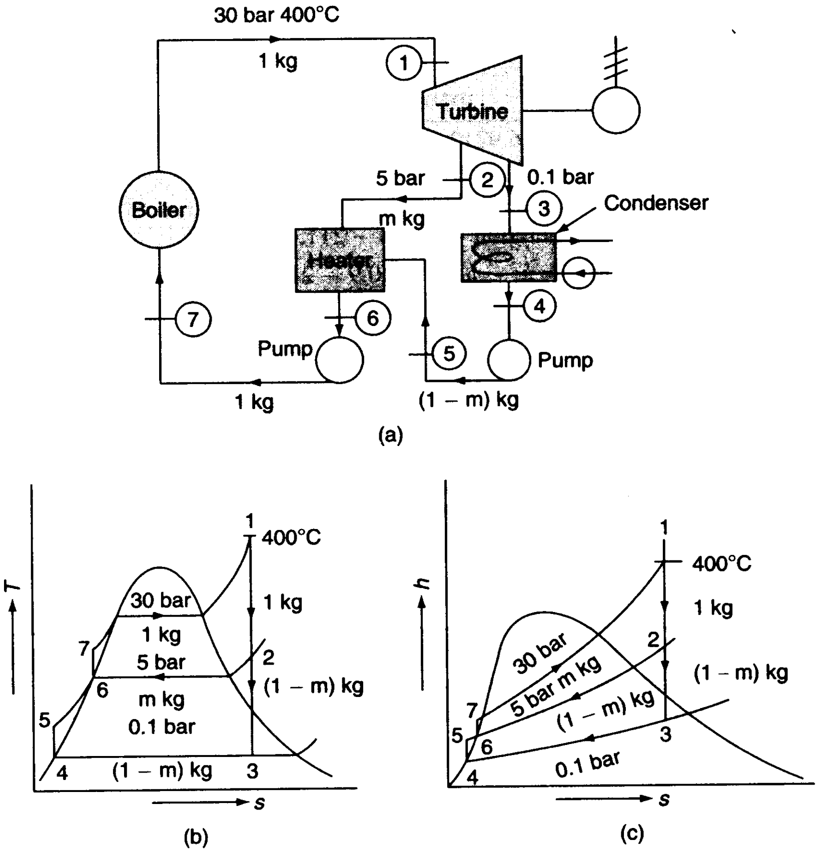
$$s_3 = 6.9212 = s_{f0.1\text{ bar}} + x_{3s} s_{fg0.1\text{ bar}}$$
$$= 0.6493 + x_3 7.5009$$

$\therefore \quad x_3 = \dfrac{6.2719}{7.5009} = 0.836$

$\therefore \quad h_3 = 191.83 + 0.836 \times 2392.8 = 2192.2$ kJ/kg

Since pump work is neglected

$$h_4 = 191.83 \text{ kJ/kg} = h_5$$
$$h_6 = 640.23 \text{ kJ/kg} = h_7$$



[illegible]

Energy balance for the heater gives

$$m(h_2 - h_6) = (1 - m)(h_6 - h_5)$$

$$m(2796 - 640.23) = (1 - m)(640.23 - 191.83)$$

$$2155.77\, m = 548.4 - 548.4\, m$$

$$\therefore \quad m = \frac{548.4}{2704.17} = 0.203 \text{ kg}$$

$$\therefore \quad W_T = (h_1 - h_2) + (1 - m)(h_2 - h_3)$$

$$= (3230.9 - 2796) + 0.797\,(2796 - 2192.2) = 916.13 \text{ kJ/kg}$$

$$Q_1 = h_1 - h_6 = 3230.9 - 640.23 = 2590.67 \text{ kJ/kg}$$

$$\eta_{cycle} = \frac{916.13}{2590.67} = 0.3536, \text{ or } 35.36\%$$ *Ans.* (a)

$$\text{Steam rate} = \frac{3600}{916.13} = 3.93 \text{ kg/kW h}$$ *Ans.* (b)

$$T_{m1} = \frac{h_1 - h_7}{s_1 - s_7} = \frac{2590.67}{6.9212 - 1.8607} = 511.95 \text{ K} = 238.8°\text{C}$$

$$T_{m1} \text{ (without regeneration)} = \frac{h_1 - h_4}{s_1 - s_4}$$
$$= \frac{3039.07}{6.9212 - 0.6493}$$
$$= 484.55 \text{ K}$$
$$= 211.4 \text{ °C}$$

Increase in T_{m1} due to regeneration = 238.8 − 211.4 = 27.4°C *Ans.* (b)

W_T (without regeneration) = $h_1 - h_3$ = 3230.9 − 2192.2 = 1038.7 kJ/kg

$$\text{Steam rate (without regeneration)} = \frac{3600}{1038.7} = 3.46 \text{ kg/kW h}$$

∴ Increase in steam rate due to regeneration

= 3.93 − 3.46 = 0.47 kg/kW h *Ans.* (b)

$$\eta_{cycle} \text{ (without regeneration)} = \frac{h_1 - h_3}{h_1 - h_4} = \frac{1038.7}{3039.07} = 0.3418 \text{ or } 34.18\%$$

∴ Increase in cycle efficiency due to regeneration

= 35.36 − 34.18 = 1.18% *Ans.* (c)

Example 12.6

In a steam power plant the condition of steam at inlet to the steam turbine is 20 bar and 300°C and the condenser pressure is 0.1 bar. Two feedwater heaters operate at optimum temperature. Determine: (a) the quality of steam at turbine exhaust, (b) net work per kg of steam, (c) cycle efficiency, and (d) the stream rate. Neglect pump work.

Solution From Fig. 12.19 (a),

$$h_1 = 3023.5 \text{ kJ/kg}$$
$$s_1 = 6.7664 \text{ kJ/kg K} = s_2 = s_3 = s_4$$
$$t_{sat} \text{ at 20 bar} \cong 212°\text{C}$$
$$t_{sat} \text{ at 0.1 bar} \cong 46°\text{C}$$
$$\Delta t_{OA} = 212 - 46 = 166°\text{C}$$

$$\therefore \text{ Temperature rise per heater} = \frac{166}{3} = 55°\text{C}$$

∴ Temperature at which the first heater operates

= 212 − 55 = 157°C ≅ 150°C (assumed)

Temperature at which the second heater operates = 157 − 55 = 102°C ≅ 100°C (assumed)

At 0.1 bar,

$$h_f = 191.83, h_{fg} = 2392.8, s_f = 0.6493$$
$$s_g = 8.1502$$

At 100°C,

$$h_f = 419.04, h_{fg} = 2257.0, s_f = 1.3069, s_g = 7.3549$$

At 150°C,

$$h_f = 632.20, h_{fg} = 2114.3, s_f = 1.8418, s_g = 6.8379$$
$$6.7664 = 1.8418 + x_2 \times 4.9961$$
$$\therefore \quad x_2 = 0.986$$
$$\therefore \quad h_2 = 632.2 + 0.986 \times 2114.3 = 2716.9 \text{ kJ/kg}$$

$$6.7664 = 1.3069 + x_3 \times 6.0480$$
$$x_3 = 0.903$$
$$\therefore \quad h_3 = 419.04 + 0.903 \times 2257.0 \quad \text{or} \quad h_3 = 2457.1 \text{ kJ/kg}$$
$$6.7664 = 0.6493 + x_4 \times 7.5010$$
$$x_4 = 0.816$$
$$\therefore \quad h_4 = 191.83 + 0.816 \times 2392.8 = 2144.3 \text{ kJ/kg}$$

Since pump work is neglected, $h_{10} = h_9$, $h_8 = h_7$, $h_6 = h_5$. By making an energy balance for the *hp* heater

$$(1 - m_1)(h_9 - h_8) = m_1(h_2 - h_9)$$

Rearranging

$$m_1 = \frac{h_9 - h_7}{h_2 - h_7} = \frac{213.16}{2297.86} = 0.093 \text{ kg}$$

By making an energy balance for the *lp* heater,

$$(1 - m_1 - m_2)(h_7 - h_6) = m_2(h_3 - h_7)$$
$$(1 - 0.093 - m_2)(419.04 - 191.83) = m_2(2457.1 - 419.04)$$
$$\therefore \quad m_2 = 0.091 \text{ kg}$$
$$\therefore \quad W_T = 1(h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1 - m_2)(h_3 - h_4)$$
$$= (3023.5 - 2716.9) + (1 - 0.093)(2716.9 - 2457.1)$$
$$+ (1 - 0.093 - 0.091)(2457.1 - 2144.3) = 797.48 \text{ kJ/kg}$$
$$Q_1 = h_1 - h_9 = 3023.5 - 632.2 = 2391.3 \text{ kJ/kg}$$
$$\therefore \quad \eta_{cycle} = \frac{W_T - W_P}{Q_1} = \frac{797.48}{2391.3} = 0.3334 \text{ or } 33.34\%$$

$$\text{Steam rate} = \frac{3600}{W_{net}} = \frac{3600}{797.48} = 4.51 \text{ kJ/kW h}$$

Example 12.7

Dry saturated steam at 40 *bar expands in a turbine isentropically to the condenser pressure of* 0.075 *bar. Hot gases available at* 2000 *K, and* 1 *atm pressure are used for steam generation and then exhausted at* 450 *K to the ambient atmosphere which is at* 300 *K and* 1 *atm. The heating rate provided by the gas stream is* 100 *MW. Assuming* c_p *of hot gases as* 1.1 *kJ/kg K, give an exergy balance of the plant and compare it with the energy balance, and find the second law efficiency.*

Solution

$$\dot{Q}_1 = w_g c_{p_g}(T_i - T_e) = 100 \text{ MW}$$
$$w_g = \text{mass flow rate of hot gas}$$
$$= \frac{100 \times 10^3}{1.1 \times (2000 - 450)} = 58.7 \text{ kg/s}$$

Exergy flow rate of inlet gas

$$a_{f_1} = w_g c_{p_g} T_0 \left[\frac{T_i}{T_0} - 1 - \ln\frac{T_i}{T_0}\right] = 58.7 \times 1.1 \times 300 \left[\frac{2000}{300} - 1 - \ln\frac{2000}{300}\right] = 73 \text{ MW}$$

Exergy flow rate of exhaust gas stream

$$a_{f_2} = 58.7 \times 1.1 \times 300\left[\frac{450}{300} - 1 - \ln\frac{450}{300}\right] = 1.83 \text{ MW}$$

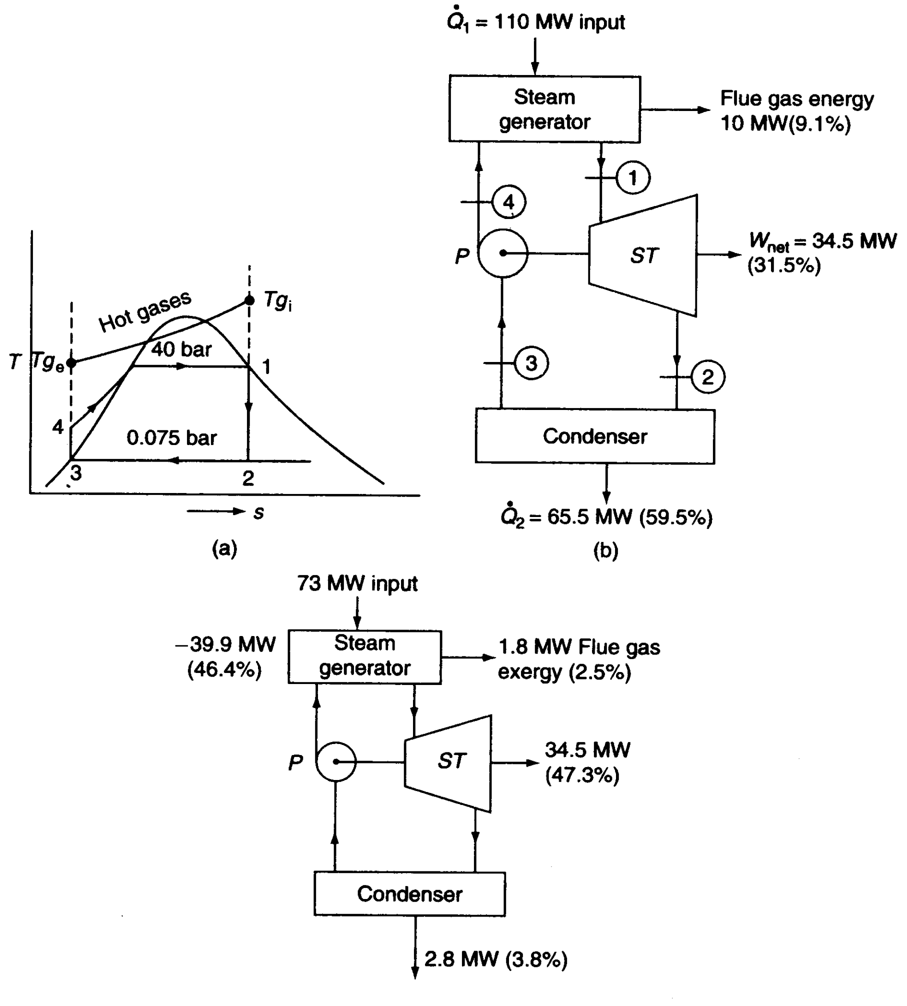


Fig. Ex. 12.7 *(a) T–s diagram, (b) Energy distribution diagram, (c) Exergy distribution diagram*

The exergy loss rate is only about $\left[\frac{1.83}{73} \times 100\right]$ or 2.5% of the initial exergy of the source gas.

The rate of exergy decrease of the gas stream,

$$a_{f_i} = \text{Exergy input rate} = 73 - 1.83 = 71.17 \cong 71.2 \text{ MW}$$

The rate of exergy increase of steam = Exergy utilization rate

$$a_{fu} = w_s\,[h_1 - h_4 - T_0(s_1 - s_4)]$$

Now,

$$h_1 = (h_g)_{40\text{ bar}} = 2801 \text{ kJ/kg}, \quad h_3 = 169 \text{ kJ/kg}$$

$$s_3 = s_4 = 0.576 \text{ kJ/kg K}, \quad h_4 = 172.8 \text{ kJ/kg}$$

$$s_1 = s_2 = 6.068 \text{ kJ/kg K}, \quad h_2 = 1890.2 \text{ kJ/kg}$$

$$W_T = h_1 - h_2 = 2801 - 1890.2 = 910.8 \text{ kJ/kg}$$

$$W_P = h_4 - h_3 = 172.8 - 169 = 3.8 \text{ kJ/kg}$$
$$Q_1 = h_1 - h_4 = 2801 - 172.8 = 2628 \text{ kJ/kg}$$
$$Q_2 = h_2 - h_3 = 1890.2 - 169 = 1721 \text{ kJ/kg}$$
$$W_{net} = W_T - W_P = Q_1 - Q_2 = 907 \text{ kJ/kg}$$
$$Q_1 = w_s \times 2628 = 100 \times 10^3 \text{ kW}$$
$$w_s = 38 \text{ kg/s}$$
$$a_{f_u} = 38\,[2801 - 172.8 - 300\,(6.068 - 0.576)] = 37.3 \text{ MW}$$

Rate of exergy destruction in the steam generator

= Rate of exergy decrease of gases – Rate of exergy increase of steam.

$$\dot{I} = a_{f_i} - a_{f_u} = 71.2 - 37.3 = 33.9 \text{ MW}$$

Rate of useful mechanical power output

$$\dot{W}_{net} = 38 \times 907 = 34.5 \text{ MW}$$

Exergy flow rate of wet steam to the condenser

$$a_{f_c} = w_s\,[h_2 - h_3 - T_0(s_2 - s_3)]$$
$$= 38\,[1890 - 169 - 300\,(6.068 - 0.576)] = 2.8 \text{ MW}$$

This is the exergy loss to the surroundings.

The energy and exergy balances are shown in Fig. Ex. 12.7 (b) and (c). The second law efficiency is given by

$$\eta_{II} = \frac{\text{Useful exergy output}}{\text{Exergy input}} = \frac{34.5}{73} = 0.473 \text{ or } 47.3\% \qquad \textit{Ans.}$$

Example 12.8

In a steam power plant, the condition of steam at turbine inlet is 80 bar, 500°C and the condenser pressure is 0.1 bar. The heat source comprises a steam of exhaust gases from a gas turbine discharging at 560°C and 1 atm pressure. The minimum temperature allowed for the exhaust gas stream is 450 K. The mass flow rate of the hot gases is such that the heat input rate to the stream cycle is 100 MW. The ambient condition is given by 300 K and 1 atm. Determine η_I, work ratio and η_{II} of the following cycles: (a) basic Rankine cycle, without superheat, (b) Rankine cycle with superheat, (c) Rankine cycle with reheat such that steam expands in the h.p. turbine until it exits as dry saturated vapour, (d) ideal regenerative cycle, with the exit temperature of the exhaust gas steam taken as 320°C, because the saturation temperature of steam at 80 bar is close to 300°C.

Solution For the first law analysis of each cycle, knowledge of the h values at each of the states indicated in Fig. Ex. 12.8 is required.

(a) *Basic Rankine cycle* (*Fig.* 12.8*a*):

By usual procedure with the help of steam tables,

$$h_1 = 2758,\ h_2 = 1817,\ h_3 = 192 \text{ and } h_4 = 200 \text{ kJ/kg}$$
$$W_T = h_1 - h_2 = 941 \text{ kJ/kg},\ W_p = h_4 - h_3 = 8 \text{ kJ/kg}$$
$$Q_1 = h_1 - h_4 = 2558 \text{ kJ/kg},\ W_{net} = 933 \text{ kJ/kg}$$
$$\eta_I = \frac{W_{net}}{Q_1} = \frac{933}{2558} = 0.365 \text{ or } 36.5\% \qquad \textit{Ans.}$$
$$\text{Work ratio} = \frac{W_T - W_P}{W_T} = \frac{933}{941} = 0.991 \qquad \textit{Ans.}$$

Power output = $\eta_I\, Q_1 = 0.365 \times 100 = 36.5$ MW

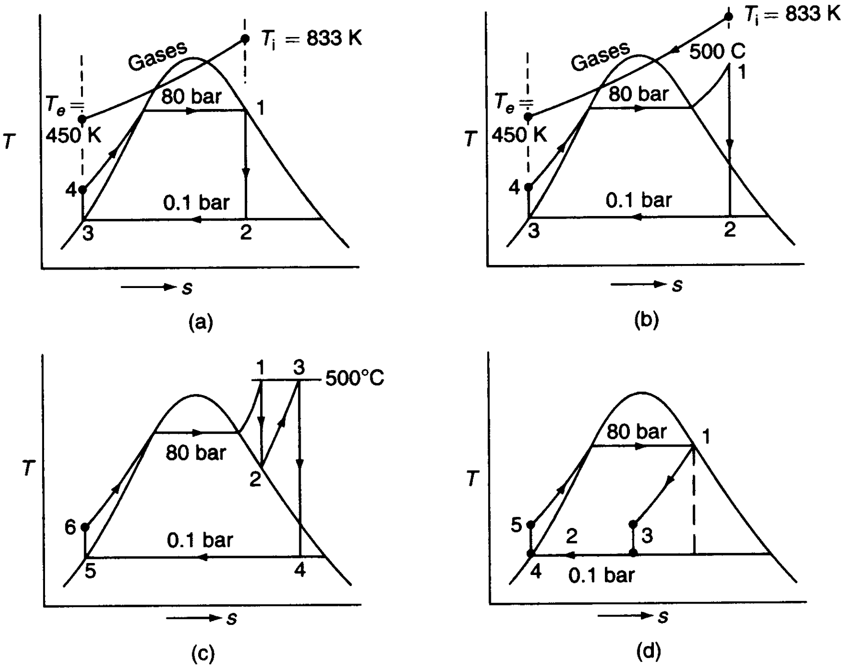


Fig. Ex. 12.8

$$\text{Exergy input rate} = w_g\, c_{p_g}\left[(T_i - T_0) - T_0 \ln\frac{T_i}{T_0}\right]$$

$$= \frac{100 \times 1000}{833 - 450}\left[(833 - 300) - 300 \ln\frac{833}{300}\right] = 59.3 \text{ MW}$$

$$\eta_{II} = \frac{36.5}{59.3} = 0.616 \text{ or } 61.6\%$$ *Ans.*

(b) *Rankine cycle with superheat* (*Fig.* 12.8*b*):

$h_1 = 3398$, $h_2 = 2130$, $h_3 = 192$ and $h_4 = 200$ kJ/kg

$W_T = 1268$ kJ/kg, $W_P = 8$ kJ/kg, $Q_1 = 3198$ kJ/kg

$$\eta_I = \frac{1260}{3198} = 0.394 \text{ or } 39.4\%$$ *Ans.*

$$\text{Work ratio} = \frac{1260}{1268} = 0.994$$ *Ans.*

Exergy input rate = 59.3 MW, $W_{net} = Q_1 \times \eta_I = 39.4$ MW

$$\eta_{II} = \frac{36.5}{59.3} = 0.664 \text{ or } 66.4\%$$ *Ans.*

Improvements in both first law and second law efficiencies are achieved with superheating. The specific work output is also increased. Therefore, conventional vapour power plants are almost always operated with some superheat.

(c) *Rankine cycle with reheat* (*Fig.* 12.8*c*):

$h_1 = 3398, h_2 = 2761, h_3 = 3482, h_4 = 2522, h_5 = 192$ and $h_6 = 200$ kJ/kg

$$W_{T1} = 637 \text{ kJ/kg}, W_{T2} = 960 \text{ kJ/kg}$$

$$W_T = 637 + 960 = 1597 \text{ kJ/kg}, W_P = 8 \text{ kJ/kg}$$

$$W_{net} = 1589 \text{ kJ/kg}, Q_1 = 3198 + 721 = 3919 \text{ kJ/kg}$$

$$\eta_I = \frac{1589}{3919} = 0.405 \text{ or } 40.5\% \quad \textit{Ans.}$$

$$\text{Work ratio} = \frac{W_{net}}{W_T} = \frac{1589}{1597} = 0.995 \quad \textit{Ans.}$$

Mechanical power output = 100 × 0.405 = 40.5 MW

Exergy input rate = 59.3 MW

$$\eta_{II} = \frac{40.5}{59.3} = 0.683 \text{ or } 68.3\% \quad \textit{Ans.}$$

Compared with basic Rankine cycle, the second law efficiency for the reheat cycle shows an increase of about 11% [(0.683 − 0.616)/0.616]. Therefore, most of the large conventional steam power plants in use today operate on the Rankine cycle with reheat.

(d) *Rankine cycle with complete regeneration* (*Fig.* 12.8*d*)

t_{sat} at 0.1 bar = 45.8°C = 318.8 K and

t_{sat} at 80 bar = 295°C = 568 K

$$\eta_I = \eta_{Carnot} = 1 - \frac{T_3}{T_1} = 1 - \frac{318.8}{568.0} = 0.439 \text{ or, } 43.9\% \quad \textit{Ans.}$$

$$Q_1 = h_1 - h_6 = 2758 - 1316 = 1442 \text{ kJ/kg}$$

$$W_{net} = Q_1 \times \eta_I = 1442 \times 0.439 = 633 \text{ kJ/kg}$$

$$W_P = 8 \text{ kJ/kg} \quad W_T = 641 \text{ kJ/kg}$$

$$\text{Work ratio} = \frac{633}{641} = 0.988 \quad \textit{Ans.}$$

Power output = 0.439 × 100 = 43.9 MW

$$\text{Exergy input rate} = \frac{100 \times 1000}{833 - 593}\left[(833 - 300) - 300 \ln\frac{833}{300}\right] = 94.583 \text{ MW} \cong 94.6 \text{ MW}$$

$$\eta_{II} = \frac{43.9}{94.6} = 0.464 \text{ or } 46.4\% \quad \textit{Ans.}$$

The second law efficiency is lower for regeneration because of the more substantial loss of exergy carried by the effluent gas stream at 593 K.

Example 12.9

A certain chemical plant requires heat from process steam at 120°C *at the rate of* 5.83 *MJ/s and power at the rate of* 1000 *kW from the generator terminals. Both the heat and power requirements are met by a back pressure turbine of* 80% *brake and* 85% *internal efficiency, which exhausts steam at* 120°C *dry saturated. All the latent heat released during condensation is utilized in the process heater. Find the pressure and temperature of steam at the inlet to the turbine. Assume* 90% *efficiency for the generator.*

Solution At 120°C, $h_{fg} = 2202.6$ kJ/kg $= h_2 - h_3$ (Fig. Ex. 12.9)

$$Q_H = w_s (h_2 - h_3) = 5.83 \text{ MJ/s}$$

$$\therefore \quad w_s = \frac{5830}{2202.6} = 2.647 \text{ kg/s}$$

$$W_{net} = \frac{1000}{0.9} \text{ kJ/s} = \text{Brake output}$$

Now $$\eta_{brake} = \frac{\text{Brake output}}{\text{Ideal output}} = \frac{(1000)/0.9}{w_s(h_1 - h_{2s})} = 0.80$$

$\therefore$ $$h_1 - h_{2s} = \frac{1000}{0.9 \times 0.8 \times 2.647} = 524.7 \text{ kJ/kg}$$

Again $$\eta_{internal} = \frac{h_1 - h_2}{h_1 - h_{2s}} = 0.85$$

$\therefore$ $$h_1 - h_2 = 0.85 \times 524.7 = 446 \text{ kJ/kg}$$

$$h_2 = h_g \text{ at } 120°\text{C} = 2706.3 \text{ kJ/kg}$$

$\therefore$ $$h_1 = 3152.3 \text{ kJ/kg}$$

$$h_{2s} = h_1 - 524.7 = 2627.6 \text{ kJ/kg} = h_f + x_{2s} h_{fg} = 503.71 + x_{2s} \times 2202.6$$

$\therefore$ $$x_{2s} = \frac{2123.89}{2202.6} = 0.964$$

$\therefore$ $$s_{2s} = s_f + x_{2s} s_{fg} = 1.5276 + 0.964 \times 5.6020 = 6.928 \text{ kJ/kg K}$$

At state 1, $$h_1 = 3152.3 \text{ kJ/kg}$$

$$s_1 = 6.928 \text{ kJ/kg K}$$

From the Mollier chart

$$p_1 = 22.5 \text{ bar}$$

$$t_1 = 360°\text{C}$$ *Ans.*

Example 12.10

A certain factory has an average electrical load of 1500 *kW and requires* 3.5 *MJ/s for heating purposes. It is proposed to install a single-extraction passout steam turbine to operate under the following conditions:*

Initial pressure 15 *bar.*
Initial temperature 300°*C.*
Condenser pressure 0.1 *bar.*

Steam is extracted between the two turbine sections at 3 *bar,* 0.96 *dry, and is isobarically cooled without subcooling in heaters to supply the heating load. The internal efficiency of the turbine (in the L.P. Section) is* 0.80 *and the efficiency of the boiler is* 0.85 *when using oil of calorific value* 44 *MJ/kg.*

If 10% *of boiler steam is used for auxiliaries calculate the oil consumption per day. Assume that the condensate from the heaters (at* 3 *bar) and that from the condenser (at* 0.1 *bar) mix freely in a separate vessel (hot well) before being pumped to the boiler. Neglect extraneous losses.*

Solution Let w_s be the flow rate of steam (kg/h) entering the turbine, and w the amount of steam extracted per hour for process heat (Fig. Ex. 12.10).

$$h_1 = 3037.3 \text{ kJ/kg}$$

$$h_2 = 561.47 + 0.96 \times 2163.8 = 2638.7 \text{ kJ/kg}$$

$$s_2 = 1.6718 + 0.96 \times 5.3201 = 6.7791 \text{ kJ/kg K} = s_{3s}$$

$$s_{3s} = 6.7791 = 0.6493 + x_{3s} \times 7.5009$$

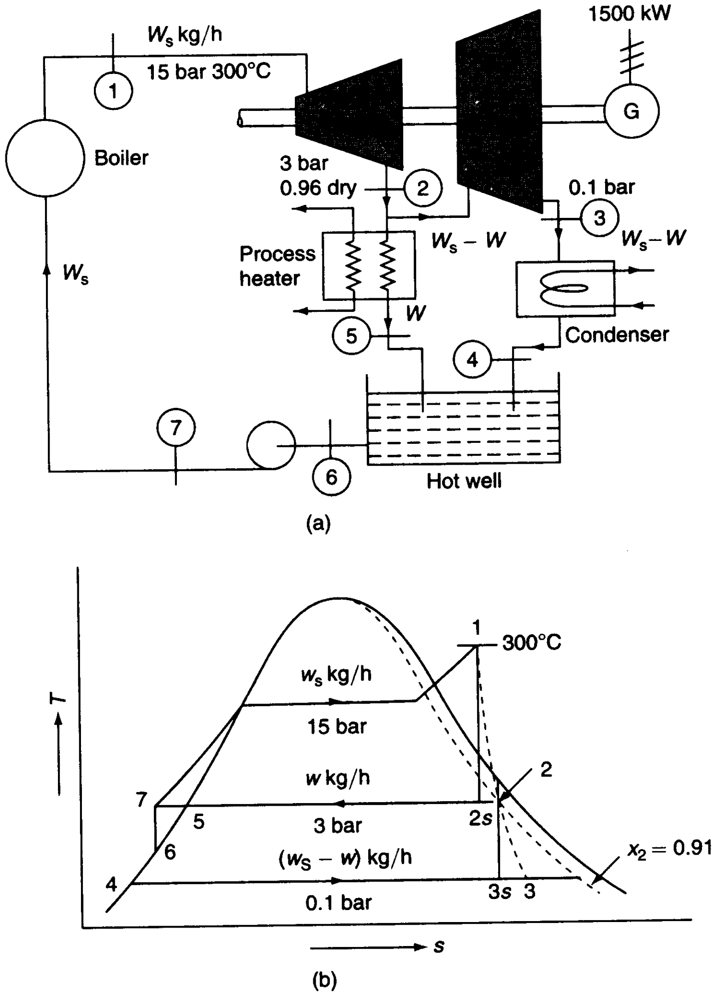


Fig. Ex. 12.10

$$x_{3s} = \frac{6.1298}{7.5009} = 0.817$$

$$h_{3s} = 191.83 + 0.817 \times 2392.8 = 2146.75 \text{ kJ/kg}$$

$$h_2 - h_{3s} = 2638.7 - 2146.75 = 491.95 \text{ kJ/kg}$$

$$h_2 - h_3 = 0.8 \times 491.95 = 393.56 \text{ kJ/kg}$$

$$\therefore \quad h_3 = 2638.7 - 393.56 = 2245.14 \text{ kJ/kg}$$

$$h_5 = 561.47 \text{ kJ/kg}, \; h_4 = 191.83 \text{ kJ/kg}$$

$$Q_H = w(h_2 - h_5) = w(2638.7 - 561.47) = 3.5 \text{ MJ/s}$$

$$\therefore \quad w = \frac{3500}{2077.23} = 1.685 \text{ kg/s}$$

Now

$$W_T = w_s(h_1 - h_2) + (w_s - w)(h_2 - h_3)$$
$$= w_s(3037.3 - 2638.7) + (w_s - 1.685) \times 393.56$$
$$= w_s \times 398.6 + w_s \times 393.56 - 663.15$$
$$= 792.16\, w_s - 663.15$$

Neglecting pump work

$$W_T = 1500 \text{ kJ/s} = 792.16\, w_s - 663.15$$

$$\therefore \quad w_s = \frac{2163.15}{792.16} = 2.73 \text{ kg/s} = 9828 \text{ kg/h}$$

By making energy balance for the hot well

$$(w_s - w)h_4 + wh_5 = w_s h_6$$
$$(2.73 - 1.685)191.83 + 1.685 \times 561.47 = 2.73 \times h_6$$
$$200.46 + 946.08 = 2.73\, h_6$$
$$\therefore \quad h_6 = 419.98 \text{ kJ/kg} \cong h_7$$

Steam raising capacity of the boiler = 1.1 w_s kg/h, since 10% of boiler steam is used for auxiliaries.

$$\therefore \quad \eta_{\text{boiler}} = \frac{1.1\, w_s\,(h_1 - h_7)}{w_f \times \text{C.V.}}$$

where w_f = fuel burning rate (kg/h)

and C.V. = calorific value of fuel = 44 MJ/kg

$$\therefore \quad 0.85 = \frac{1.1 \times 9828 \times (3037.3 - 419.98)}{w_f \times 44000}$$

or

$$w_f = \frac{1.1 \times 9828 \times 2617.32}{0.85 \times 44000} = 756.56 \text{ kg/h}$$

$$= \frac{756.56 \times 24}{1000} = 18.16 \text{ tonnes/day}$$ *Ans.*

Example 12.11

A steam turbine gets its supply of steam at 70 bar and 450°C. After expanding to 25 bar in high pressure stages, it is reheated to 420°C at the constant pressure. Next, it is expanded in intermediate pressure stages to an appropriate minimum pressure such that part of the steam bled at this pressure heats the feedwater to a temperature of 180°C. The remaining steam expands from this pressure to a condenser pressure of 0.07 bar in the low pressure stage. The isentropic efficiency of the h.p. stage is 78.5%, while that of the intermediate and l.p. stages is 83% each. From the above data (a) determine the minimum pressure at which bleeding is necessary, and sketch a line diagram of the arrangement of the plant, (b) sketch on the T–s diagram all the processes, (c) determine the quantity of steam bled per kg of flow at the turbine inlet, and (d) calculate the cycle efficiency. Neglect pump work.

Solution Figure Ex. 12.11 gives the flow and *T–s* diagrams of the plant. It would be assumed that the feedwater heater is an open heater. Feedwater is heated to 180°C. So p_{sat} at 180°C ≅ 10 bar is the pressure at which the heater operates.

Therefore, the pressure at which bleeding is necessary is 10 bar. *Ans.* (a).

From the Mollier chart

$$h_1 = 3285,\ h_{2s} = 3010,\ h_3 = 3280,\ h_{4s} = 3030 \text{ kJ/kg}$$

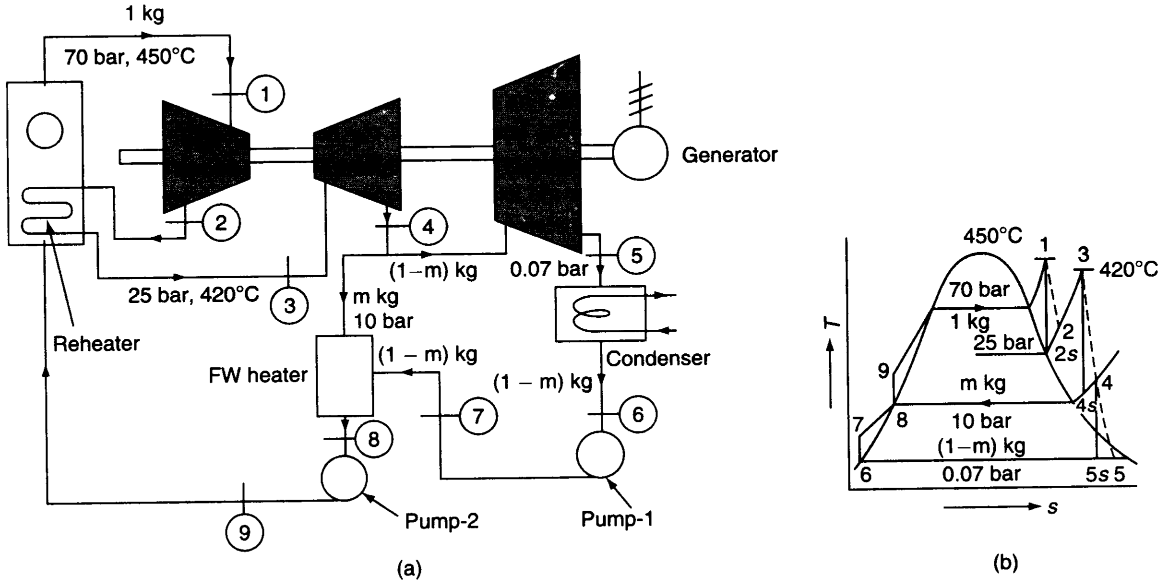


Fig. Ex. 12.11

$$h_3 - h_4 = 0.83\,(h_3 - h_{4s}) = 0.83 \times 250 = 207.5 \text{ kJ/kg}$$

$$h_4 = 3280 - 207.5 = 3072.5 \text{ kJ/kg}$$

$$h_{5s} = 2225 \text{ kJ/kg}$$

$$h_4 - h_5 = 0.83(h_4 - h_{5s}) = 0.83 \times 847.5 = 703.4 \text{ kJ/kg}$$

$$\therefore \quad h_5 = 3072.5 - 703.4 = 2369.1 \text{ kJ/kg}$$

$$h_6 = 162.7 \text{ kJ/kg}$$

$$h_8 = 762.81 \text{ kJ/kg}$$

$$h_1 - h_2 = 0.785\,(h_1 - h_{2s}) = 0.785 \times 275 = 215.9 \text{ kJ/kg}$$

$$h_2 = 3285 - 215.9 = 3069.1 \text{ kJ/kg}$$

Energy balance for the heater gives

$$m \times h_4 + (1 - m)h_7 = 1 \times h_8$$

$$m \times 3072.5 + (1 - m) \times 162.7 = 1 \times 762.81$$

$$m = \frac{600.11}{2909.8} = 0.206 \text{ kg/kg steam flow at turbine inlet.}$$ *Ans.* (c)

$$\therefore \quad \eta_{\text{cycle}} = \frac{(h_1 - h_2) + (h_3 - h_4) + (1 - m)(h_4 - h_5)}{(h_1 - h_8) + (h_3 - h_2)}$$

$$= \frac{215.9 + 207.5 + 0.794 \times 703.4}{2522.2 + 210.9}$$

$$= \frac{981.9}{2733.1}\ 0.3592 \text{ or } 35.92\%$$ *Ans.* (d)

Example 12.12

A binary-vapour cycle operates on mercury and steam. Saturated mercury vapour at 4.5 *bar is supplied to the mercury turbine, from which it exhausts at* 0.04 *bar. The mercury condenser generates saturated steam at* 15 *bar which is expanded in a steam turbine to* 0.04 *bar. (a) Find the overall efficiency of the cycle. (b) If* 50,000 *kg/h of steam flows through the steam turbine, what is the flow through the mercury turbine? (c) Assuming that all processes are reversible, what is the useful work done in the binary vapour cycle for the specified steam flow? (d) If the steam leaving the mercury condenser is superheated to a temperature of* 300°*C in a superheater located in the mercury boiler, and if the internal efficiencies of the mercury and steam turbines are* 0.85 *and* 0.87 *respectively, calculate the overall efficiency of the cycle. The properties of saturated mercury are given below*

p (*bar*)	*t*(°*C*)	h_f	h_g	s_f	s_g	v_f	v_g
		(*kJ/kg*)		(*kJ/kg K*)		(m^3/*kg*)	
4.5	450	62.93	355.98	0.1352	0.5397	79.9×10^{-6}	0.068
0.04	216.9	29.98	329.85	0.0808	0.6925	76.5×10^{-6}	5.178

Solution The cycle is shown in Fig. Ex. 12.12.

For the mercury cycle, $h_a = 355.98$ kJ/kg

$$s_a = 0.5397 \text{ kJ/kg K} = s_b = s_f + x_b\, s_{fg}$$
$$= 0.0808 + x_b (0.6925 - 0.0808)$$

$$\therefore \quad x_b = \frac{0.4589}{0.6117} = 0.75$$

$$h_b = 29.98 + 0.75 \times 299.87$$
$$= 254.88 \text{ kJ/kg}$$
$$(W_T)_m = h_a - h_b$$
$$= 355.98 - 254.88 = 101.1 \text{ kJ/kg}$$
$$(W_P)_m = h_d - h_c$$
$$= 76.5 \times 10^{-6} \times 4.46 \times 100$$
$$= 3.41 \times 10^{-2} \text{ kJ/kg}$$
$$\therefore W_{net} = 101.1 \text{ kJ/kg}$$
$$Q_1 = h_a - h_d$$
$$= 355.98 - 29.98 = 326 \text{ kJ/kg}$$
$$\therefore \eta_m = \frac{W_{net}}{Q_1} = \frac{101.1}{326} = 0.31 \text{ or } 31\%$$

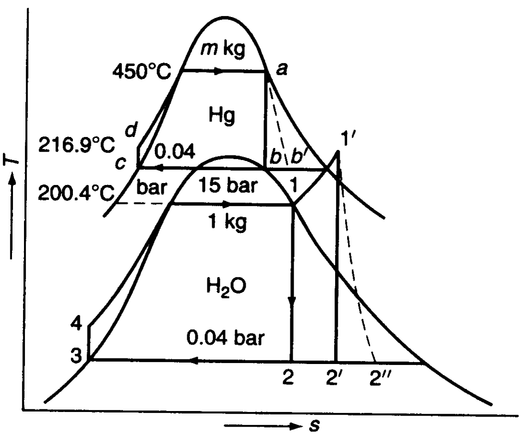


Fig. Ex. 12.12

For the steam cycle

$$h_1 = 2792.2 \text{ kJ/kg}$$
$$s_1 = 6.4448 \text{ kJ/kg K} = s_2 = s_f + x_2\, s_{fg2} = 0.4226 + x_2 (8.4746 - 0.4226)$$
$$x_2 = \frac{6.0222}{8.0520} = 0.748$$
$$h_2 = 121.46 + 0.748 \times 2432.9 = 1941.27 \text{ kJ/kg}$$

$$(W_T)_{St} = h_1 - h_2 = 2792.2 - 1941.27 = 850.93 \text{ kJ/kg}$$
$$(W_P)_{St} = h_4 - h_3 = 0.001 \times 14.96 \times 100 = 1.496 \text{ kJ/kg} \cong 1.5 \text{ kJ/kg}$$
$$h_4 = 121.46 + 1.5 = 122.96 \text{ kJ/kg}$$
$$Q_1 = h_1 - h_4 = 2792.2 - 122.96 = 2669.24 \text{ kJ/kg}$$
$$(W_{net})_{St} = 850.93 - 1.5 = 849.43 \text{ kJ/kg}$$
$$\therefore \quad \eta_{st} = \frac{W_{net}}{Q_1} = \frac{849.43}{2669.24} = 0.318 \text{ or } 31.8\%$$

Overall efficiency of the binary cycle would be

$$\eta_{overall} = \eta_m + \eta_{St} - \eta_m \cdot \eta_{St} = 0.31 + 0.318 - 0.31 \times 0.318$$
$$= 0.5294 \text{ or } 52.94\%$$ *Ans.* (a)

$\eta_{overall}$ can also be determined in, the following way:

By writing the energy balance for a mercury condenser-steam boiler

$$m(h_b - h_c) = 1(h_1 - h_4)$$

where m is the amount of mercury circulating for 1 kg of steam in the bottom cycle.

$$\therefore \quad m = \frac{h_1 - h_4}{h_b - h_c} = \frac{2669.24}{254.88 - 29.88} = \frac{2669.24}{224.90} = 11.87 \text{ kg}$$
$$(Q_1)_{total} = m(h_a - h_d) = 11.87 \times 326 = 3869.6 \text{ kJ/kg}$$
$$(W_T)_{total} = m(h_a - h_b) + (h_1 - h_2) = 11.87 \times 101.1 - 850.93 = 2051 \text{ kJ/kg}$$

$(W_P)_{total}$ may be neglected

$$\therefore \quad \eta_{overall} = \frac{W_{net}}{Q_1} = \frac{2051}{3869.6} = 0.53 \text{ or } 53\%$$

If 50,000 kg/h of steam flows through the steam turbine, the flow rate of mercury w_m would be

$$w_m = 50{,}000 \times 11.87 = 59.35 \times 10^4 \text{ kg/h}$$ *Ans.* (b)

$$(W_T)_{total} = 2051 \times 50{,}000 = 10255 \times 10^4 \text{ kJ/h} = 0.2849 \times 10^5 \text{ kW} = 28.49 \text{ MW}$$ *Ans.* (c)

Considering the efficiencies of turbines

$$(W_T)_m = h_a - h'_b = 0.85 \times 101.1 = 84.95 \text{ kJ/kg}$$
$$\therefore \quad h'_b = 355.98 - 85.94 = 270.04 \text{ kJ/kg}$$
$$\therefore \quad m'(h'_b - h'_c) = (h_1 - h_4)$$
$$\therefore \quad m' = \frac{2669.24}{240.06} = 11.12 \text{ kg}$$
$$(Q_1)_{total} = m'(h_a - h_d) + 1(h'_1 - h_1)$$
$$= 11.12 \times 326 + (3037.3 - 2792.2) = 3870.22 \text{ kJ/kg}$$
$$s'_1 = 6.9160 = 0.4226 + x'_2(8.4746 - 0.4226)$$
$$x'_2 = \frac{6.4934}{8.0520} = 0.806$$
$$h'_2 = 121.46 + 0.806 \times 2432.9 = 2082.38 \text{ kJ/kg}$$
$$(W_T)_{St} = h_i - h_2'' = 0.87(3037.3 - 2082.38) = 830.78 \text{ kJ/kg}$$
$$(W_T)_{total} = 11.12 \times 85.94 + 830.78 = 1786.43 \text{ kJ/kg}$$

Pump work is neglected.

$$\eta_{overall} = \frac{1786.43}{3870.22} = 0.462 \text{ or } 46.2\%$$ *Ans.* (d)

Review Questions

12.1 What are the four basic components of a steam power plant?

12.2 What is the reversible cycle that represents the simple steam power plant? Draw the flow, p–v, T–s and h–s diagrams of this cycle.

12.3 What do you understand by steam rate and heat rate? What are their units?

12.4 Why is Carnot cycle not practicable for a steam power plant?

12.5 What do you understand by the mean temperature of heat addition?

12.6 For a given T_2, show how the Rankine cycle efficiency depends on the mean temperature of heat addition.

12.7 What is metallurgical limit?

12.8 Explain how the quality at turbine exhaust gets restricted.

12.9 How are the maximum temperature and maximum pressure in the Rankine cycle fixed?

12.10 When is reheating of steam recommended in a steam power plant? How does the reheat pressure get optimized?

12.11 What is the effect of reheat on (a) the specific output, (b) the cycle efficiency, (c) steam rate, and (d) heat rate, of a steam power plant?

12.12 Give the flow and T–s diagrams of the ideal regenerative cycle. Why is the efficiency of this cycle equal to Carnot efficiency? Why is this cycle not practicable?

12.13 What is the effect of regeneration on the (a) specific output, (b) mean temperature of heat addition, (c) cycle efficiency, (d) steam rate and (e) heat rate of a steam power plant?

12.14 How does the regeneration of steam carnotize the Rankine cycle?

12.15 What are open and closed heaters? Mention their merits and demerits.

12.16 Why is one open feedwater heater used in a steam plant? What is it called?

12.17 How are the number of heaters and the degree of regeneration get optimized?

12.18 Draw the T–s diagram of an ideal working fluid in a vapour power cycle.

12.19 Discuss the desirable characteristics of a working fluid in a vapour power cycle.

12.20 Mention a few working fluids suitable in the high temperature range of a vapour power cycle.

12.21 What is a binary vapour cycle?

12.22 What are topping and bottoming cycles?

12.23 Show that the overall efficiency of two cycles coupled in series equals the sum of the individual efficiencies minus their product.

12.24 What is a cogeneration plant? What are the thermodynamic advantages of such a plant?

12.25 What is a back pressure turbine? What are its applications?

12.26 What is the biggest loss in a steam plant? How can this loss be reduced?

12.27 What is a pass-out turbine? When is it used?

12.28 Define the following: (a) internal work, (b) internal efficiency, (c) brake efficiency (d) mechanical efficiency, and (e) boiler efficiency.

12.29 Express the overall efficiency of a steam plant as the product of boiler, turbine, generator and cycle efficiencies.

Problems

12.1 For the following steam cycles, find (a) W_T in kJ/kg (b) W_P in kJ/kg, (c) Q_1 in kJ/kg, (d) cycle efficiency, (e) steam rate in kg/kW h, and (f) moisture at the end of the turbine process. Show the results in tabular form with your comments.

Boiler outlet	Condenser pressure	Type of cycle
10 bar, saturated	1 bar	Ideal Rankine cycle
-do-	-do-	Neglect W_P
-do-	-do-	Assume 75% pump and turbine efficiency
-do-	0.1 bar	Ideal Rankine cycle

(Continued)

10 bar, 300°C	-do-	-do-
150 bar, 600°C	-do-	-do-
-do-	-do-	Reheat to 600°C at maximum intermediate pressure to limit end moisture to 15%
-do-	-do-	-do- but with 85% turbine efficiency
10 bar, saturated	0.1 bar	Isentropic pump process ends on saturated liquid line
-do-	-do-	-do- but with 80% machine efficiencies
-do-	-do-	Ideal regenerative cycle
-do-	-do-	Single open heater at 110°C
-do-	-do-	Two open heaters at 90°C and 135°C
-do-	-do-	-do- but the heaters are closed heaters

12.2 A geothermal power plant utilizes steam produced by natural means underground. Steam wells are drilled to tap this steam supply which is available at 4.5 bar and 175°C. The steam leaves the turbine at 100 mm Hg absolute pressure. The turbine isentropic efficiency is 0.75. Calculate the efficiency of the plant. If the unit produces 12.5 MW, what is the steam flow rate?

12.3 A simple steam power cycle uses solar energy for the heat input. Water in the cycle enters the pump as a saturated liquid at 40°C, and is pumped to 2 bar. It then evaporates in the boiler at this pressure, and enters the turbine as saturated vapour. At the turbine exhaust the conditions are 40°C and 10% moisture. The flow rate is 150 kg/h. Determine (a) the turbine isentropic efficiency, (b) the net work output (c) the cycle efficiency, and (d) the area of solar collector needed if the collectors pick up 0.58 kW/m^2. *Ans.* (a) 0.767, (b) 15.51 kW, (c) 14.7%, (d) 182.4 m^2

12.4 In a reheat cycle, the initial steam pressure and the maximum temperature are 150 bar and 550°C respectively. If the condenser pressure is 0.1 bar and the moisture at the condenser inlet is 5%, and assuming ideal processes, determine (a) the reheat pressure, (b) the cycle efficiency, and (c) the steam rate. *Ans.* 13.5 bar, 43.6%, 2.05 kg/kW h

12.5 In a nuclear power-plant heat is transferred in the reactor to liquid sodium. The liquid sodium is then pumped to a heat exchanger where heat is transferred to steam. The steam leaves this heat exchanger as saturated vapour at 55 bar, and is then superheated in an external gas-fired superheater to 650°C. The steam then enters the turbine, which has one extraction point at 4 bar, where steam flows to an open feedwater heater. The turbine efficiency is 75% and the condenser temperature is 40°C. Determine the heat transfer in the reactor and in the superheater to produce a power output of 80 MW.

12.6 In a reheat cycle, steam at 500°C expands in a h.p. turbine till it is saturated vapour. It is reheated at constant pressure to 400°C and then expands in a l.p. turbine to 40°C. If the maximum moisture content at the turbine exhaust is limited to 15%, find (a) the reheat pressure, (b) the pressure of steam at the inlet to the h.p. turbine, (c) the net specific work output, (d) the cycle efficiency, and (e) the steam rate. Assume all ideal processes.

What would have been the quality, the work output, and the cycle efficiency without the reheating of steam? Assume that the other conditions remain the same.

12.7 A regenerative cycle operates with steam supplied at 30 bar and 300°C, and condenser pressure of 0.08 bar. The extraction points for two heaters (one closed and one open) are at 3.5 bar and 0.7 bar respectively. Calculate the thermal efficiency of the plant, neglecting pump work.

12.8 The net power output of the turbine in an ideal reheat-regenertive cycle is 100 MW. Steam enters the high-pressure (H.P.) turbine at 90 bar, 550°C. After expansion to 7 bar, some of the steam goes to an open heater and the balance is reheated to 400°C, after which it expands to 0.07 bar. (a) What is the steam flow rate to the H.P. turbine? (b) What is the total pump work? (c) Calculate the cycle efficiency. (d) If there is a 10°C rise in the temperature of the cooling water, what is the rate of flow of the cooling water in the condenser? (e) If the velocity of the steam flowing from the turbine to the condenser is limited to a maximum of 130 m/s, find the diameter of the connecting pipe.

12.9 A mercury cycle is superposed on the steam cycle operating between the boiler outlet condition of 40 bar, 400°C and the condenser temperature of 40°C. The heat released by mercury condensing at 0.2 bar is used to impart the latent heat of vaporization to the water in the steam cycle. Mercury enters the mercury turbine as saturated vapour at 10 bar. Compute (a) kg of mercury circulated per kg of water, and (b) the efficiency of the combined cycle.

The property values of saturated mercury are given below

p (bar)	*t* (°C)	h_f	h_g	s_f	s_g	v_f	v_g
		(kJ/kg)		(kJ/kg K)		(m³/kg)	
10	515.5	72.23	363.0	0.1478	0.5167	80.9×10^{-6}	0.0333
0.2	277.3	38.35	336.55	0.0967	0.6385	77.4×10^{-6}	1.163

12.10 In an electric generating station, using a binary vapour cycle with mercury in the upper cycle and steam in the lower, the ratio of mercury flow to steam flow is 10:1 on a mass basis. At an evaporation rate of 1,000,000 kg/h for the mercury, its specific enthalpy rises by 356 kJ/kg in passing through the boiler. Superheating the steam in the boiler furnace adds 586 kJ to the steam specific enthalpy. The mercury gives up 251.2 kJ/kg during condensation, and the steam gives up 2003 kJ/kg in its condenser. The overall boiler efficiency is 85%. The combined turbine mechanical and generator efficiencies are each 95% for the mercury and steam units. The steam auxiliaries require 5% of the energy generated by the units. Find the overall efficiency of the plant.

12.11 A sodium-mercury-steam cycle operates between 1000°C and 40°C. Sodium rejects heat at 670°C to mercury. Mercury boils at 24.6 bar and rejects heat at 0.141 bar. Both the sodium and mercury cycles are saturated. Steam is formed at 30 bar and is superheated in the sodium boiler to 350°C. It rejects heat at 0.08 bar. Assume isentropic expansions, no heat losses, and no regeneration and neglect pumping work. Find (a) the amounts of sodium and mercury used per kg of steam, (b) the heat added and rejected in the composite cycle per kg steam, (c) the total work done per kg steam. (d) the efficiency of the composite cycle, (e) the efficiency of the corresponding Carnot cycle, and (f) the work, heat added, and efficiency of a supercritical pressure steam (single fluid) cycle operating at 250 bar and between the same temperature limits.

For mercury, at 24.6 bar, $h_g = 366.78$ kJ/kg
$s_g = 0.48$ kJ/kg K and at 0.141 bar, $s_f = 0.09$
and $s_g = 0.64$ kJ/kg K, $h_f = 36.01$
and $h_g = 330.77$ kJ/kg
For sodium, at 1000°C, $h_g = 4982.53$ kJ/kg
At turbine exhaust, $h = 3914.85$ kJ/kg
At 670°C, $h_f = 745.29$ kJ/kg

For a supercritical steam cycle, the specific enthalpy and entropy at the turbine inlet may be computed by extrapolation from the steam tables.

12.12 A textile factory requires 10,000 kg/h of steam for process heating at 3 bar saturated and 1000 kW of power, for which a back pressure turbine of 70% internal efficiency is to be used. Find the steam condition required at the inlet to the turbine.

12.13 A 10,000 kW steam turbine operates with steam at the inlet at 40 bar, 400°C and exhausts at 0.1 bar. Ten thousand kg/h of steam at 3 bar are to be extracted for process work. The turbine has 75% isentropic efficiency throughout. Find the boiler capacity required.

12.14 A 50 MW steam plant built in 1935 operates with steam at the inlet at 60 bar, 450°C and exhausts at 0.1 bar, with 80% turbine efficiency. It is proposed to scrap the old boiler and put in a new boiler and a topping turbine of efficiency 85% operating with inlet steam at 180 bar, 500°C. The exhaust from the topping turbine at 60 bar is reheated to 450°C and admitted to the old turbine. The flow rate is just sufficient to produce the rated output from the old turbine. Find the improvement in efficiency with the new set up. What is the additional power developed?

12.15 A steam plant operates with an initial pressure at 20 bar and temperature 400°C, and exhausts to a heating system at 2 bar. The condensate from the heating system is returned to the boiler plant at 65°C, and the heating system utilizes for its intended purpose 90% of the energy transferred from the steam it receives. The turbine efficiency is 70%. (a) What fraction of the energy supplied to the steam plant serves a useful purpose? (b) If two separate steam plants had been set up to produce the same useful energy, one to generate heating steam at 2 bar, and the other to generate power through a cycle working between 20 bar, 400°C and 0.07 bar, what fraction of the energy supplied would have served a useful purpose?
Ans. 91.2%, 64.5%

12.16 In a nuclear power plant saturated steam at 30 bar enters a h.p. turbine and expands isentropically to a pressure at which its quality is 0.841. At this pressure the steam is passed through a moisture separator which removes all the liquid. Saturated vapour leaves the separator and is expanded isentropically to 0.04 bar in l.p. turbine, while the saturated liquid leaving the separator is returned via a feed pump to the boiler. The condensate leaving the condenser at 0.04 bar is also returned to the boiler via a second feed pump. Calculate the cycle efficiency and turbine outlet quality taking into account the feed pump term. Recalculate the same quantities for a cycle with the same boiler and condenser pressures but without moisture separation. *Ans.* 35.5%, 0.824; 35%; 0.716

12.17 The net power output of an ideal regenerative-reheat steam cycle is 80 MW. Steam enters the h.p. turbine at 80 bar, 500°C and expands till it becomes saturated vapour. Some of the steam then goes to an open feedwater heater and the balance is reheated to 400°C, after which it expands in the l.p. turbine to 0.07 bar. Compute (a) the reheat pressure, (b) the steam flow rate to the h.p. turbine, and (c) the cycle efficiency. Neglect pump work. *Ans.* 6.5 bar, 58.4 kg/s, 43.7%

12.18 Figure P 12.18 shows the arrangement of a steam plant in which steam is also required for an industrial heating process. The steam leaves boiler *B* at 30 bar, 320°C and expands in the H.P. turbine to 2 bar, the efficiency of the H.P. turbine being 75%. At this point one half of the steam passes to the process heater *P* and the remainder enters separator *S* which removes all the moisture. The dry steam enters the L.P. turbine at 2 bar and expands to the condenser pressure 0.07 bar, the efficiency of the L.P. turbine being 70%. The drainage from the separator mixes with the condensate from the process heater and the combined flow enters the hotwell *H* at 80°C. Traps are provided at the exist from *P* and *S*. A pump extracts the condensate from condenser *C* and this enters

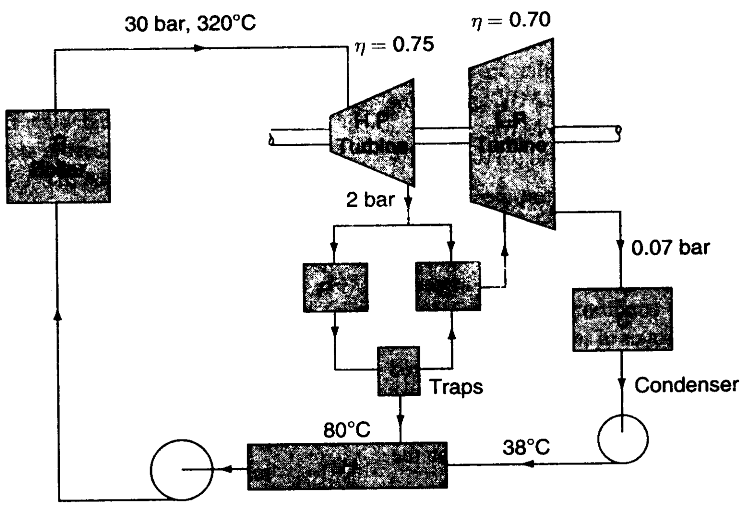


Fig. P 12.18

the hotwell at 38°C. Neglecting the feed pump work and radiation loss, estimate the temperature of water leaving the hotwell which is at atmospheric pressure. Also calculate, as percentage of heat transferred in the boiler, (a) the heat transferred in the process heater, and (b) the work done in the turbines.

12.19 In a combined power and process plant the boiler generates 21,000 kg/h of steam at a pressure of 17 bar, and temperature 230°C. A part of the steam goes to a process heater which consumes 132.56 kW, the steam leaving the process heater 0.957 dry at 17 bar being throttled to 3.5 bar. The remaining steam flows through a H.P. turbine which exhausts at a pressure of 3.5 bar. The exhaust steam mixes with the process steam before entering the L.P. turbine which develops 1337.5 kW. At the exhaust the pressure is 0.3 bar, and the steam is 0.912 dry. Draw a line diagram of the plant and determine (a) the steam quality at the exhaust from the H.P. turbine, (b) the power developed by the H.P. turbine, and (c) the isentropic efficiency of the H.P. turbine.

Ans. (a) 0.96, (b) 1125 kW, (c) 77%

12.20 In a cogeneration plant, the power load is 5.6 MW and the heating load is 1.163 MW. Steam is generated at 40 bar and 500°C and is expanded isentropically through a turbine to a condenser at 0.06 bar. The heating load is supplied by extracting steam from the turbine at 2 bar which condensed in the process heater to saturated liquid at 2 bar and then pumped back to the boiler. Compute (a) the steam generation capacity of the boiler in tonnes/h, (b) the heat input to the boiler in MW, and (c) the heat rejected to the condenser in MW.

Ans. (a) 19.07 t/h, (b) 71.57 MW, and (c) 9.607 MW

12.21 Steam is supplied to a pass-out turbine at 35 bar, 350°C and dry saturated process steam is required at 3.5 bar. The low pressure stage exhausts at 0.07 bar and the condition line may be assumed to be straight (the condition line is the locus passing through the states of steam leaving the various stages of the turbine). If the power required is 1 MW and the maximum process load is 1.4 kW, estimate the maximum steam flow through the high and low pressure stages. Assume that the steam just condenses in the process plant.

Ans. 1.543 and 1.182 kg/s

12.22 Geothermal energy from a natural geyser can be obtained as a continuous supply of steam 0.87 dry at 2 bar and at a flow rate of 2700 kg/h. This is utilized in a mixed-pressure cycle to augment the superheated exhaust from a high pressure turbine of 83% internal efficiency, which is supplied with 5500 kg/h of steam at 40 bar and 500°C. The mixing process is adiabatic and the mixture is expanded to a condenser pressure of 0.10 bar in a low pressure turbine of 78% internal efficiency. Determine the power output and the thermal efficiency of the plant. *Ans.* 1745 kW, 35%

12.23 In a study for a space project it is thought that the condensation of a working fluid might be possible at −40°C. A binary cycle is proposed, using Refrigerant-12 as the low temperature fluid, and water as the high temperature fluid. Steam is generated at 80 bar, 500°C and expands in a turbine of 81% isentropic efficiency to 0.06 bar, at which pressure it is condensed by the generation of dry saturated refrigerant vapour at 30°C from saturated liquid at −40°C. The isentropic efficiency of the R-12 turbine is 83%. Determine the mass ratio of R-12 to water and the efficiency of the cycle. Neglect all losses. *Ans.* 10.86; 44.4%

12.24 Steam is generated at 70 bar, 500°C and expands in a turbine to 30 bar with an isentropic efficiency of 77%. At this condition it is mixed with twice its mass of steam at 30 bar, 400°C. The mixture then expands with an isentropic efficiency of 80% to 0.06 bar. At a point in the expansion where the pressure is 5 bar, steam is bled for feedwater heating in a direct contact heater, which raises the feedwater to the saturation temperature of the bled steam. Calculate the mass of steam bled per kg of high pressure steam and the cycle efficiency. Assume that the L.P. expansion condition line is straight. *Ans.* 0.53 kg; 31.9%

12.25 An ideal steam power plant operates between 70 bar, 550°C and 0.075 bar. It has seven feedwater heaters. Find the optimum pressure and temperature at which each of the heaters operate.

12.26 In a reheat cycle steam at 550°C expands in an h.p. turbine till it is saturated vapour. It is reheated at constant pressure to 400°C and then expands in a l.p. turbine to 40°C. If the moisture content at turbine exhaust is given to be 14.67%, find (a) the reheat pressure, (b) the pressure of steam at inlet to the h.p. turbine, (c) the net work output per kg, and (d) the cycle efficiency. Assume all processes to be ideal.

Ans. (a) 20 bar, (b) 200 bar, (c) 1604 kJ/kg, (d) 43.8%

12.27 In a reheat steam cycle, the maximum steam temperature is limited to 500°C. The condenser pressure is 0.1 bar and the quality at turbine exhaust is 0.8778. Had there been no reheat, the exhaust quality would have been 0.7592. Assuming ideal processes, determine (a) reheat pressure, (b) the boiler pressure, (c) the cycle efficiency, and (d) the steam rate. *Ans.* (a) 30 bar, (b) 150 bar, (c) 50.51%, (d) 1.9412 kg/kWh

12.28 In a cogeneration plant, steam enters the h.p. stage of a two-stage turbine at 1 MPa, 200°C and leaves it at 0.3 MPa. At this point some of the steam is bled off and passed through a heat exchanger which it leaves as saturated liquid at 0.3 MPa. The remaining steam expands in the l.p. stage of the turbine to 40 kPa. The turbine is required to produce a total power of 1 MW and the heat exchanger to provide a heating rate of 500 KW. Calculate the required mass flow rate of steam into the h.p. stage of the turbine. Assume (a) steady condition throughout the plant, (b) velocity and gravity terms to be negligible, (c) both turbine stages are adiabatic with isentropic efficiencies of 0.80. *Ans.* 2.457 kg/s

12.29 A steam power plant is designed to operate on the basic Rankine cycle. The heat input to the boiler is at the rate of 50 MW. The H_2O exits the condenser as saturated liquid and exits the boiler as saturated vapour. The pressure of steam at boiler exit is 120 bar and the condenser pressure is 0.04 bar. The heat input to the boiler is provided by a steady stream of hot gases initially at 2200 K and 1 atm. The hot gases exhaust at 600 K and 1 atm to the surroundings which are at 600 K and 1 atm. Taking the c_p of hot gases as 1.1 kJ/kgK, determine (a) the cycle efficiency, (b) the work output, (c) the power output (in MW), (d) the required mass flow rate of steam (in kg/h), (e) the specific steam consumption (in kg/kWh), (f) the mass flow rate (in kg/h) of the stream of hot gases, (g) the exergy flux (in MW) of the inlet gases, (h) the exergy loss rate (in MW) with the exhaust gases, (i) the exergy consumption (in MW) in the steam generation process, (j) the exergy consumption (in MW) in the condensation process, (i) the second law efficiency.

Ans. (f) 1.20×10^5 kg/h, (g) 41.5 MW, (h) 3.14 MW, (i) 16.9 MW, (j) 1.40 MW, (k) 48.3%

12.30 In a cogeneration plant the steam generator provides 10^6 kg/h of steam at 80 bar, 480°C, of which 4×10^5 kg/h is extracted between the first and second turbine stages at 10 bar and diverted to a process heating load. Condensate returns from the process heating load at 9.5 bar, 120°C and is mixed with liquid exiting the lower-pressure pump at 9.5 bar. The entire flow is then pumped to the steam generator pressure. Saturated liquid at 0.08 bar leaves the condenser. The turbine stages and the pumps operate with isentropic efficiencies of 86% and 80%, respectively. Determine (a) the heating load, in kJ/h, (b) the power developed by the turbine, in kW, (c) the rate of heat transfer in the steam generator, in kJ/h. *Ans.* (a) 9.529×10^8 kJ/h, (b) 236,500 kW, (c) 3.032×10^9 kJ/h

C H A P T E R

13 Gas Power Cycles

Here gas is the working fluid. It does not undergo any phase change. Engines operating on gas cycles may be either cyclic or non-cyclic. Hot air engines using air as the working fluid operate on a closed cycle. Internal combustion engines where the combustion of fuel takes place inside the engine cylinder are non-cyclic heat engines.

13.1 CARNOT CYCLE (1824)

The Carnot cycle (Fig. 13.1) has been discussed in Chapters 6 and 7. It consists of:

Two reversible isotherms and two reversible adiabatics. If an ideal gas is assumed as the working fluid. Then for 1 kg of gas,

$$Q_{1-2} = RT_1 \ln \frac{v_2}{v_1}; \quad W_{1-2} = RT_1 \ln \frac{v_2}{v_1} \quad Q_{2-3} = 0; \quad W_{2-3} = -c_v (T_3 - T_2)$$

$$Q_{3-4} = RT_2 \ln \frac{v_4}{v_3}; \quad W_{3-4} = RT_2 \ln \frac{v_4}{v_3} \quad Q_{4-1} = 0; \quad W_{4-1} = -c_v(T_1 - T_4)$$

$\therefore$

$$\sum_{\text{cycle}} đQ = \sum_{\text{cycle}} đW$$

Now

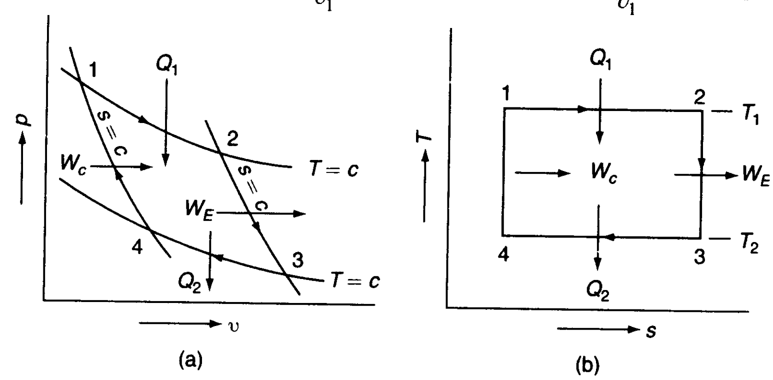
$$\frac{v_2}{v_3} = \left(\frac{T_2}{T_1}\right)^{1/(\gamma-1)} \quad \text{and} \quad \frac{v_1}{v_4} = \left(\frac{T_2}{T_1}\right)^{1/(\gamma-1)}$$

$\therefore$

$$\frac{v_2}{v_3} = \frac{v_1}{v_4} \text{ or } \frac{v_2}{v_1} = \frac{v_3}{v_4}$$

Therefore

$$Q_1 = \text{Heat added} = RT_1 \ln \frac{v_2}{v_1} \quad W_{net} = Q_1 - Q_2 = R \ln \frac{v_2}{v_1} \cdot (T_1 - T_2)$$



Carnot cycle

$$\therefore \qquad \eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{T_1 - T_2}{T_1} \tag{13.1}$$

The large back work ($W_c = W_{4-1}$) is a big drawback for the Carnot gas cycle, as in the case of the Carnot vapour cycle.

13.2 STIRLING CYCLE (1827)

The Stirling cycle (Fig. 13.2) consists of:

Two reversible isotherms and two reversible isochores. For 1 kg of ideal gas

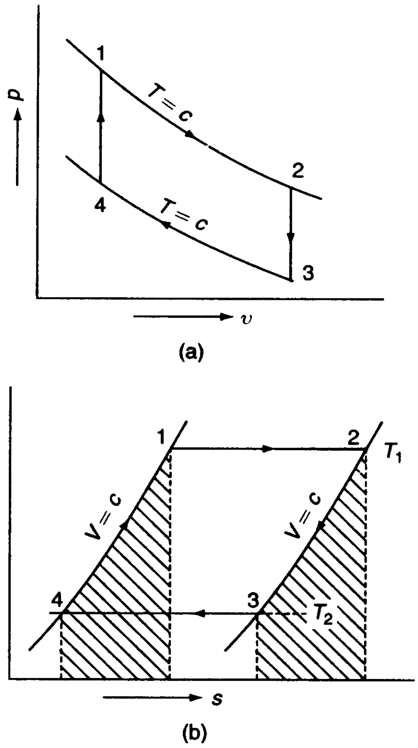
$$Q_{1-2} = W_{1-2} = RT_1 \ln \frac{v_2}{v_1} \qquad Q_{2-3} = -c_v(T_2 - T_1);\ W_{2-3} = 0$$

$$Q_{3-4} = W_{3-4} = -RT_2 \ln \frac{v_3}{v_4} \qquad Q_{4-1} = c_v(T_1 - T_2);\ W_{4-1} = 0$$

Due to heat transfers at constant volume processes, the efficiency of the Stirling cycle is less than that of the Carnot cycle. However, if a regenerative arrangement is used such that $Q_{2-3} = Q_{4-1}$, i.e. the area under 2 – 3 is equal to the area under 4 – 1, then the cycle efficiency becomes

$$\eta = \frac{RT_1 \ln \frac{v_2}{v_1} - RT_2 \ln \frac{v_3}{v_4}}{RT_1 \ln \frac{v_2}{v_1}} = \frac{T_1 - T_2}{T_1} \tag{13.2}$$

So, the regenerative Stirling cycle has the same efficiency as the Carnot cycle.



Stirling cycle

13.3 ERICSSON CYCLE (1850)

The Ericsson cycle (Fig. 13.3) is made up of:

Two reversible isotherms and two reversible isobars.

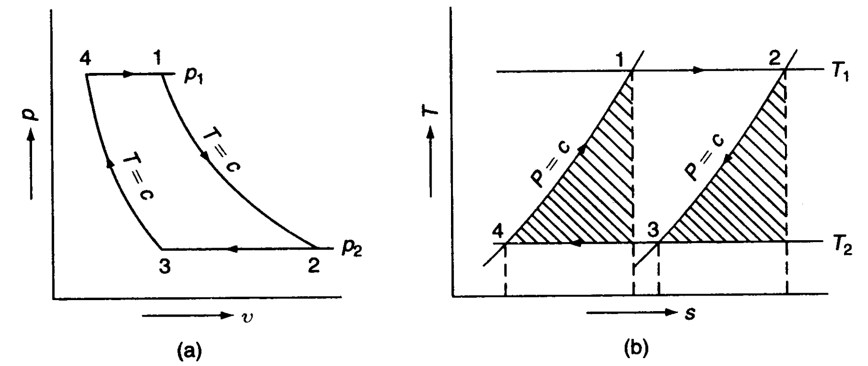
For 1 kg of ideal gas

$$Q_{1-2} = W_{1-2} = RT_1 \ln \frac{p_1}{p_2} \qquad Q_{2-3} = c_p(T_2 - T_1);\ W_{2-3} = p_2(v_3 - v_2) = R(T_2 - T_1)$$

$$Q_{3-4} = W_{3-4} = -RT_2 \ln \frac{p_1}{p_2} \qquad Q_{4-1} = c_p(T_1 - T_4);\ W_{4-1} = p_1(v_1 - v_4) = R(T_1 - T_2)$$

Since part of the heat is transferred at constant pressure and part at constant temperature, the efficiency of the Ericsson cycle is less than that of the Carnot cycle. But with ideal regeneration, $Q_{2-3} = Q_{4-1}$ so that all the heat is added from the external source at T_1 and all the heat is rejected to an external sink at T_2, the efficiency of the cycle becomes equal to the Carnot cycle efficiency, since

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{RT_2 \ln \frac{p_1}{p_2}}{RT_1 \ln \frac{p_1}{p_2}} = 1 - \frac{T_2}{T_1} \tag{13.3}$$



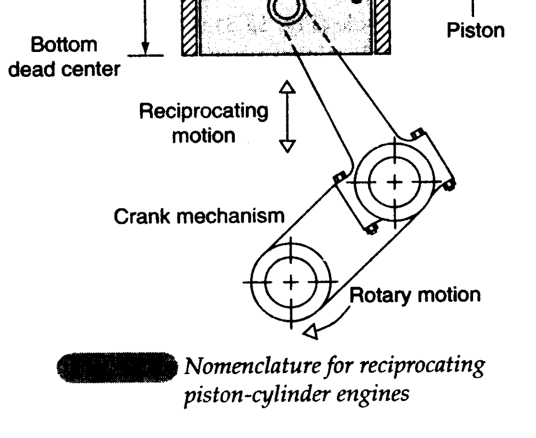
Ericsson cycle

The regenerative, Stirling and Ericsson cycles have the same efficiency as the Carnot cycle, but much less back work. Hot air engines working on these cycles have been successfully operated. But it is difficult to transfer heat to a gas at high rates, since the gas film has a very low thermal conductivity. So there has not been much progress in the development of hot air engines. However, since the cost of internal combustion engine fuels is getting excessive, these may find a field of use in the near future.

13.4 AIR STANDARD CYCLES

Internal combustion engines (Fig. 13.4) in which the combustion of fuel occurs in the engine cylinder itself are non-cyclic heat engines. The temperature due to the evolution of heat because of the combustion of fuel inside the cylinder is so high that the cylinder is cooled by water circulation around it to avoid rapid deterioration. The working fluid, the fuel-air mixture, undergoes permanent chemical change due to combustion, and the products of combustion after doing work are thrown out of the engine, and a fresh charge is taken. So the working fluid does not undergo a complete thermodynamic cycle.

To simplify the analysis of I.C. engines, *air standard cycles* are conceived. In an air standard cycle, a certain mass of air operates in a complete thermodynamic cycle, where heat is added and rejected with external heat reservoirs, and all the processes in the cycle are reversible. Air is assumed to behave as an ideal gas, and its specific heats are assumed to be constant. These air standard cycles are so conceived that they correspond to the operations of internal combustion engines.



Nomenclature for reciprocating piston-cylinder engines

13.5 OTTO CYCLE (1876)

One very common type of internal combustion engines is the *Spark Ignition (S.I.) engine* used inautomobiles. The Otto cycle is the air standard cycle of such an engine. The sequence of precesses in the elementary operation of the S.I. engine is given below, whith reference to Fig. 13.5(a,b) where the sketches of the engine and the indicatior diagram are given.

Process 1–2, *Intake.* The inlet valve is open, the piston moves to the right, admitting fuel-air mixture into the cylinder at constant pressure.

Process 2–3, *Compression.* Both the valves are closed, the piston compresses the combustible mixture to the minimum volume.

Process 3–4, *Combustion.* The mixture is then ignited by means of a spark, combustion takes place, and there is an increase in temperature and pressure.

Process 4–5, *Expansion.* The products of combustion do work on the piston which moves to the right, and the pressure and temperature of the gases decrease.

Process 5–6, *Blow-down.* The exhaust valve opens, and the pressure drops to the initial pressure.

Process 6–1, *Exhaust.* With the exhaust valve open, the piston moves inwards to expel the combustion products from the cylinder at constant pressure.

The series of processes as described above constitute a *mechanical cycle*, and not a thermodynamic cycle. The cycle is completed in four strokes of the piston.

Figure 13.5 (c) shows the air standard cycle (Otto cycle) corresponding to the above engine. It consists of:

Two reversible adiabatics and two reversible isochores.

Air is compressed in process 1 – 2 reversibly and adiabatically. Heat is then added to air reversibly at constant volume in process 2 – 3. Work is done by air in expanding reversibly and adiabatically in process 3 – 4. Heat is then rejected by air reversibly at constant volume in process 4 – 1, and the system (air) comes back to

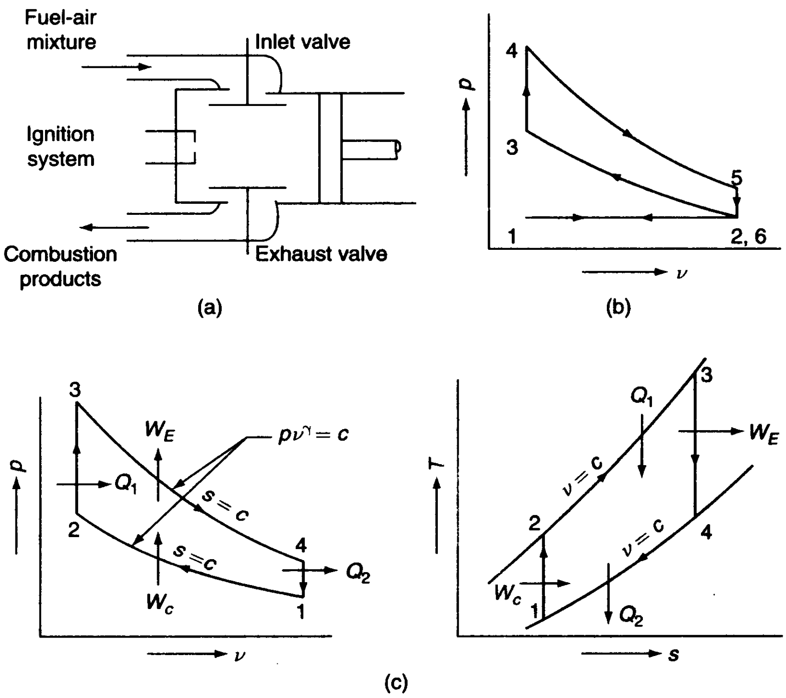


Fig. 13.5 (a) S.I. engine (b) indicator diagram (c) otto cycle

its initial state. Heat transfer processes have been substituted for the combustion and blow-down processes of the engine. The intake and exhaust processes of the engine cancel each other.

Let m be the fixed mass of air undergoing the cycle of operations as described above.

Heat supplied $Q_1 = Q_{2-3} = mc_v (T_3 - T_2)$

Heat rejected $Q_2 = Q_{4-1} = mc_v (T_4 - T_1)$

$$\text{Efficiency } \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{mc_v(T_4 - T_1)}{mc_v(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2} \tag{13.4}$$

Process 1–2,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

Process 3–4,

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

$\therefore$

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_3}{T_2} = \frac{T_4}{T_1}$$

$$\frac{T_3}{T_2} - 1 = \frac{T_4}{T_1} - 1$$

$\therefore$

$$\frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1}$$

$\therefore$ From Eq. (13.4), $\eta = 1 - \left(\frac{v_2}{v_1}\right)^{\gamma-1}$

or

$$\eta_{\text{otto}} = 1 - \frac{1}{r_k^{\gamma-1}} \tag{13.5}$$

where r_k is called the compression ratio and given by

$$r_k = \frac{\text{Volume at the beginning of compression}}{\text{Volume at the end of compression}} = \frac{V_1}{V_2} = \frac{v_1}{v_2}$$

The efficiency of the air standard Otto cycle is thus a function of the compression ratio only. The higher the compression ratio, the higher the efficiency. It is independent of the temperature levels at which the cycle operates. The compression ratio cannot, however, be increased beyond a certain limit, because of a noisy and destructive combustion phenomenon, known as detonation. It also depends upon the fuel, the engine design, and the operating conditions.

13.6 DIESEL CYCLE (1892)

The limitation on compression ratio in the S.I. engine can be overcome by compressing air alone, instead of the fuel-air mixture, and then injecting the fuel into the cylinder in spray form when combustion is desired. The temperature of air after compression must be high enough so that the fuel sprayed into the hot air burns spontaneously. The rate of burning can, to some extent, be controlled by the rate of injection of fuel. An engine operating in this way is called a *compression ignition (C.I.) engine*. The sequence of processes in the elementary operation of a C.I. engine, shown in Fig. 13.6, is given below.

Process 1–2, *intake*. The air valve is open. The piston moves out admitting air into the cylinder at constant pressure.

Process 2–3, *Compression*. The air is then compressed by the piston to the minimum volume with all the valves closed.

Process 3–4, *Fuel injection and combustion*. The fuel valve is open, fuel is sprayed into the hot air, and combustion takes place at constant pressure.

Process 4–5, *Expansion*. The combustion products expand, doing work on the piston which moves out to the maximum volume.

Process 5–6, *Blow-down*. The exhaust valve opens, and the pressure drops to the initial pressure.

Pressure 6–1, *Exhaust*. With the exhaust valve open, the piston moves towards the cylinder cover driving away the combustion products from the cylinder at constant pressure.

The above processes constitute an engine cycle, which is completed in four strokes of the piston or two revolution of the crank shaft.

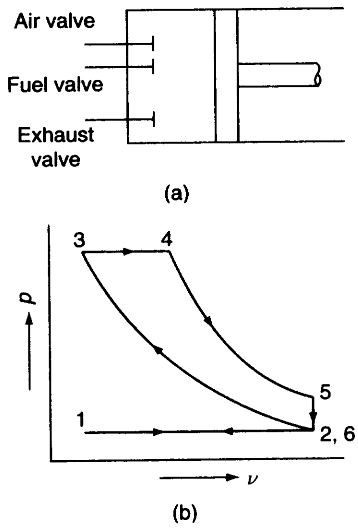


Fig. 13.6 (a) C.I. engine, (b) indicator diagram

Figure 13.7 shows the air standard cycle, called the *Diesel cycle*, corresponding to the C.I. engine, as described above. The cycle is composed of:

Two reversible adiabatics, one reversible isobar, and one reversible isochore.

Air is compressed reversibly and adiabatically in process 1 – 2. Heat is then added to it from an external source reversibly at constant pressure in process 2 – 3. Air then expands reversibly and adiabatically in process 3 – 4. Heat is rejected reversibly at constant volume in process 4 – 1, and the cycle repeats itself.

For m kg of air in the cylinder, the efficiency analysis of the cycle can be made as given below.

Heat supplied, $$Q_1 = Q_{2-3} = mc_p (T_3 - T_2)$$

Heat rejected, $$Q_2 = Q_{4-1} = mc_v (T_4 - T_1)$$

Efficiency $$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{mc_v(T_4 - T_1)}{mc_p(T_3 - T_2)}$$

$$\therefore \quad \eta = 1 - \frac{T_4 - T_1}{\gamma(T_3 - T_2)} \tag{13.6}$$

The efficiency may be expressed in terms of any two of the following three ratios

Compression ratio, $$r_k = \frac{V_1}{V_2} = \frac{v_1}{v_2}$$

Expansion ratio, $$r_e = \frac{V_4}{V_3} = \frac{v_4}{v_3}$$

Cut-off ratio, $$r_c = \frac{V_3}{V_2} = \frac{v_3}{v_2}$$

It is seen that $$r_k = r_e \cdot r_c$$

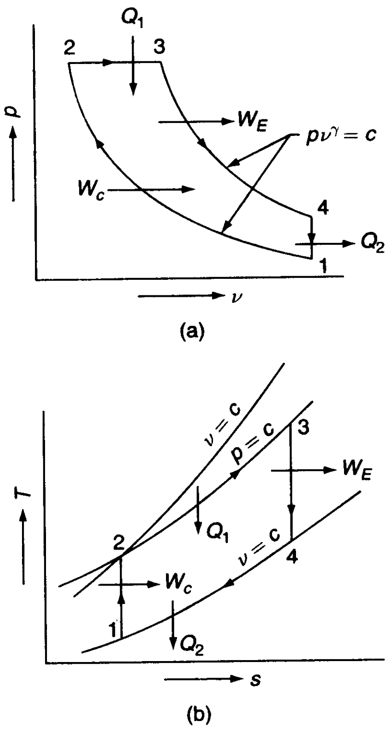


Fig. 13.7 Diesel cycle

Process 3–4

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma-1} = \frac{1}{r_e^{\gamma-1}} \qquad T_4 = T_3 \frac{r_c^{\gamma-1}}{r_k^{\gamma-1}}$$

Process 2–3

$$\frac{T_2}{T_3} = \frac{p_2 v_2}{p_3 v_3} = \frac{v_2}{v_3} = \frac{1}{r_c}$$

$\therefore$

$$T_2 = T_3 \cdot \frac{1}{r_c}$$

Process 1–2

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1} = \frac{1}{r_k^{\gamma-1}}$$

$\therefore$

$$T_1 = T_2 \cdot \frac{1}{r_k^{\gamma-1}} = \frac{T_3}{r_c} \cdot \frac{1}{r_k^{\gamma-1}}$$

Substituting the values of T_1, T_2 and T_4 in the expression of efficiency (Eq. 13.6)

$$\eta = 1 - \frac{T_3 \cdot \frac{r_c^{\gamma-1}}{r_k^{\gamma-1}} - \frac{T_3}{r_c}\frac{1}{r_k^{\gamma-1}}}{\gamma\left(T_3 - T_3 \cdot \frac{1}{r_c}\right)} \qquad \eta_{\text{Diesel}} = 1 - \frac{1}{\gamma} \cdot \frac{1}{r_k^{\gamma-1}} \cdot \frac{r_c^{\gamma}-1}{r_c - 1} \tag{13.7}$$

As $r_c > 1$, $\frac{1}{\gamma}\left(\frac{r_c^{\gamma}-1}{r_c-1}\right)$ is also greater than unity. Therefore, the efficiency of the Diesel cycle is less than that of the Otto cycle for the same compression ratio.

13.7 LIMITED PRESSURE CYCLE, MIXED CYCLE OR DUAL CYCLE

The air standard Diesel cycle does not simulate exactly the pressure-volume variation in an actual compression ignition engine, where the fuel injection is started before the end of compression stroke. A closer approximation is the limited pressure cycle in which some part of heat is added to air at constant volume, and the remainder at constant pressure.

Figure 13.8 shows the $p - v$ and $T - s$ diagrams of the dual cycle. Heat is added reversibly, partly at constant volume (2 – 3) and partly at constant pressure (3 – 4).

Heat supplied $\qquad Q_1 = mc_v (T_3 - T_2) + mc_p (T_4 - T_3)$

Heat rejected $\qquad Q_2 = mc_v (T_5 - T_1)$

Efficiency

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{mc_v (T_5 - T_1)}{mc_v (T_3 - T_2) + mc_v (T_4 - T_3)} = 1 - \frac{T_5 - T_1}{(T_3 - T_2) + \gamma (T_4 - T_3)} \tag{13.8}$$

The efficiency of the cycle can be expressed in terms of the following ratios

Compression ratio, $\qquad r_k = \frac{V_1}{V_2}$

Expansion ratio, $\qquad r_e = \frac{V_5}{V_4}$

Cut-off ratio, $\qquad r_c = \frac{V_4}{V_3}$

Constant volume pressure ratio, $r_p = \frac{p_3}{p_2}$

It is seen, as before that

$$r_k = r_c \cdot r_e$$

or $$r_e = \frac{r_k}{r_c}$$

Process 3 – 4

$$r_c = \frac{V_4}{V_3} = \frac{T_4 p_3}{p_4 T_3} = \frac{T_4}{T_3}$$

$$T_3 = \frac{T_4}{r_c}$$

Process 2 – 3

$$\frac{p_2 V_2}{T_2} = \frac{p_3 V_3}{T_3}$$

$$T_2 = T_3 \frac{p_2}{p_3} = \frac{T_4}{r_p \cdot r_c}$$

Process 1 – 2

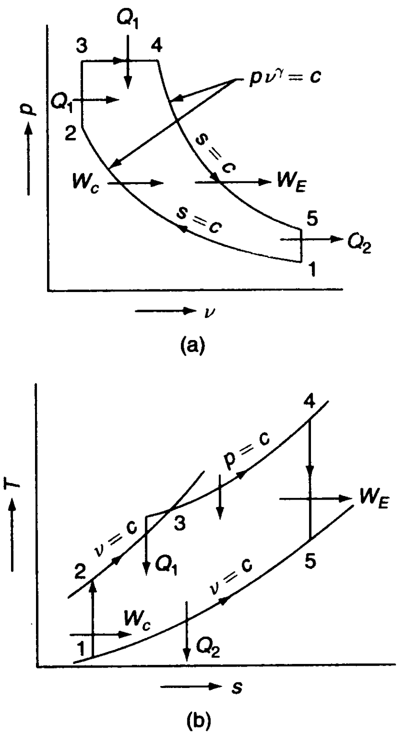
$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1} = \frac{1}{r_k^{\gamma-1}}$$

$$\therefore \quad T_1 = \frac{T_4}{r_p \cdot r_c \cdot r_k^{\gamma-1}}$$

Process 4 – 5

$$\frac{T_5}{T_4} = \left(\frac{v_4}{v_5}\right)^{\gamma-1} = \frac{1}{r_e^{\gamma-1}}$$

$$\therefore \quad T_5 = T_4 \cdot \frac{r_c^{\gamma-1}}{r_k^{\gamma-1}}$$



Limited pressure cycle, mixed cycle or dual cycle

Substituting the values of T_1, T_2, T_3, and T_5 in the expression of efficiency (Eq. 13.8).

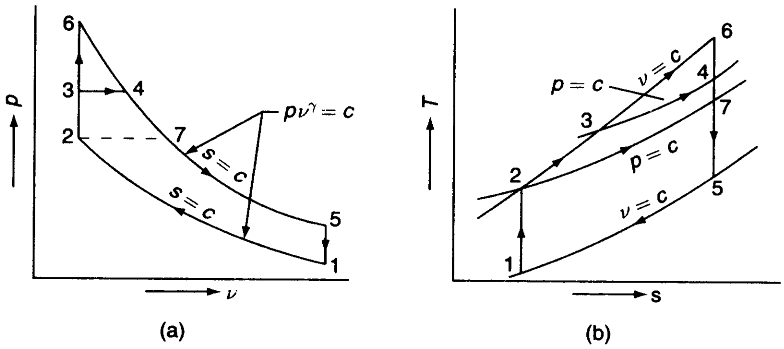
$$\eta = 1 - \frac{T_4 \cdot \frac{r_c^{\gamma-1}}{r_k^{\gamma-1}} - \frac{T_4}{r_p \cdot r_c \cdot r_k^{\gamma-1}}}{\left(\frac{T_4}{r_c} - \frac{T_4}{r_p \cdot r_c}\right) + \gamma\left(T_4 - \frac{T_4}{r_c}\right)}$$

$$\therefore \quad \eta_{Dual} = 1 - \frac{1}{r_k^{\gamma-1}} \frac{r_p \cdot r_c^{\gamma} - 1}{r_p - 1 + \gamma r_p (r_c - 1)} \qquad (13.9)$$

13.8 COMPARISON OF OTTO, DIESEL, AND DUAL CYCLES

The three cycles can be compared on the basis of either the same compression ratio or the same maximum pressure and temperature.

Figure 13.9 shows the comparison of Otto, Diesel, and Dual cycles for the same compression ratio and heat rejection. Here



Comparison of otto, diesel and dual cycles for the same compression ratio

1-2-6-5 —Otto cycle
1-2-7-5 —Diesel cycle
1-2-3-4-5 —Dual cycle

For the same Q_2, the higher the Q_1, the higher is the cycle efficiency. In the $T - s$ diagram, the area under 2 – 6 represents Q_1 for the Otto cycle, the area under 2 – 7 represents Q_1 for the Diesel cycle, and the area under 2-3-4 represents Q_1 for the Dual cycle. Therefore, for the same r_k and Q_2

$$\eta_{\text{Otto}} > \eta_{\text{Dual}} > \eta_{\text{Diesel}}$$

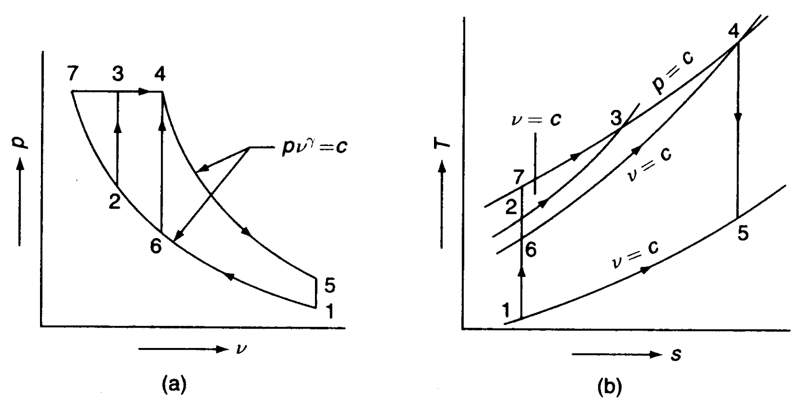
Figure 13.10 shows a comparison of the three air standard cycles for the same maximum pressure and temperature (state 4), the heat rejection being also the same. Here

1-6-4-5 —Otto cycle
1-7-4-5 —Diesel cycle
1-2-3-4-5 —Dual cycle

Q_1 is represented by the area under 6–4 for the Otto cycle, by the area under 7–4 for the Diesel cycle and by the area under 2-3-4 for the Dual cycle in the T–s plot, Q_2 being the same.

$$\therefore \quad \eta_{\text{Diesel}} > \eta_{\text{Dual}} > \eta_{\text{Otto}}$$

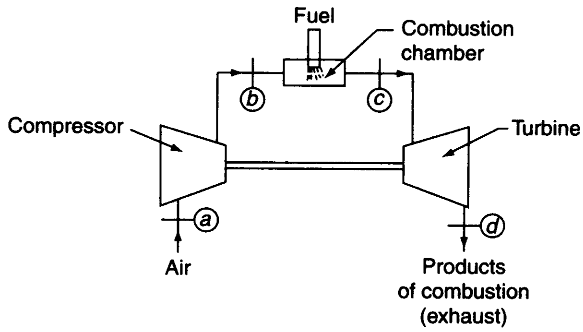
This comparison is of greater significance, since the Diesel cycle would definitely have a higher compression ratio than the Otto cycle.



Comparison of otto, diesel and dual cycles for the same maximum pressure and temperature

13.9 BRAYTON CYCLE

A simple gas turbine power plant is shown in Fig. 13.11. Air is first compressed adiabatically in process $a - b$, it then enters the combustion chamber where fuel is injected and burned essentially at constant pressure in process $b - c$, and then the products of combustion expand in the turbine to the ambient pressure in process $c - d$ and are thrown out to the surroundings. The cycle is open. The state diagram on the $p - v$ coordinates is shown in Fig. 13.12. Open cycles are used in aircraft, automotive (buses and trucks) and industrial gas turbine installations.



A simple gas turbine plant

The Brayton cycle is the air standard cycle for the gas turbine power plant. Here air is first compressed reversibly and adiabatically, heat is added to it reversibly at constant pressure, air expands in the turbine reversibly and adiabatically, and heat is then rejected from the air reversibly at constant pressure to bring it to the initial state. The Brayton cycle, therefore, consists of:

Two reversible isobars and two reversible adiabatics.

The flow, $p - v$, and $T - s$ diagrams are shown in Fig. 13.13. For m kg of air

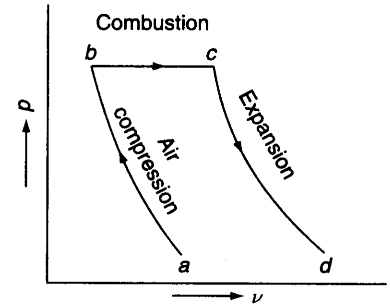
$$Q_1 = \text{heat supplied} = mc_p(T_3 - T_2)$$

$$Q_2 = \text{heat rejected} = mc_p(T_4 - T_1)$$

$\therefore$ Cycle efficiency,

$$\eta = 1 - \frac{Q_2}{Q_1}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad (13.10)$$

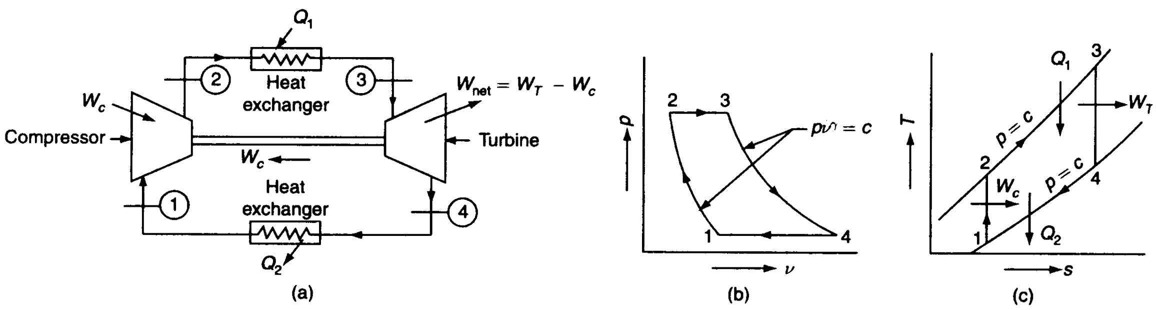


State diagram of a gas turbine plant on p–v plot

Now

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = \frac{T_3}{T_4} \text{ (Since } p_2 = p_3, \text{ and } p_4 = p_1)$$

$$\therefore \quad \frac{T_4}{T_1} - 1 = \frac{T_3}{T_2} - 1$$



(a–c) Brayton cycle, (d) comparison of Rankine cycle and braytom cycle, both operating between the same pressures p_1 and p_2

or
$$\frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{(\gamma-1)/\gamma} = \left(\frac{v_2}{v_1}\right)^{\gamma-1}$$

If r_k = compression ratio = v_1/v_2 the efficiency becomes (from Eq. 13.10)

$$\eta = 1 - \left(\frac{v_2}{v_1}\right)^{\gamma-1}$$

or
$$\eta_{\text{Brayton}} = 1 - \frac{1}{r_k^{\gamma-1}} \qquad (13.11)$$

$$\text{Work ratio} = \frac{W_T - W_C}{W_T} = \frac{Q_1 - Q_2}{W_T}$$

If r_p = pressure ratio = p_2/p_1 the efficiency may be expressed in the following form also

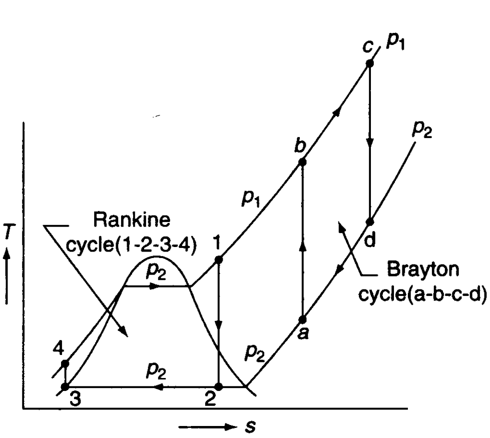


Fig. 13.13 (Continued)

$$\eta = 1 - \left(\frac{p_1}{p_2}\right)^{(\gamma-1)/\gamma}$$

or
$$\eta_{\text{Brayton}} = 1 - \frac{1}{(r_p)^{(\gamma-1)/\gamma}} \qquad (13.12)$$

The efficiency of the Brayton cycle, therefore, depends upon either the compression ratio or the pressure ratio. For the same compression ratio, the Brayton cycle efficiency is equal to the Otto cycle efficiency.

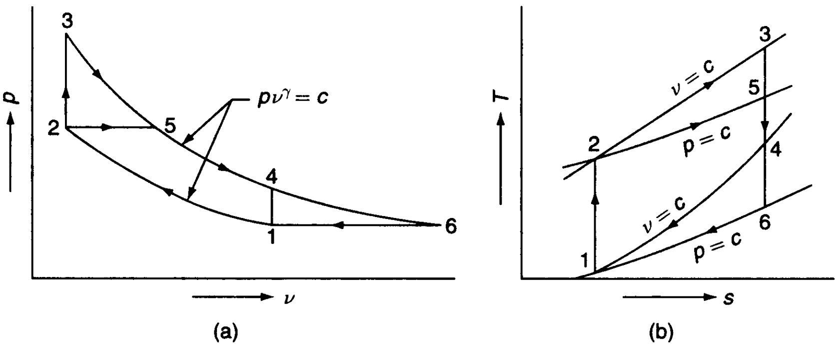
A closed cycle gas turbine plant (Fig. 13.13) is used in a gas-cooled nuclear reactor plant, where the source is a high temperature gas-cooled reactor (HTGR) supplying heat from nuclear fission directly to the working fluid (a gas).

Both Rankine cycle and Brayton cycle consist of two reversible isobars and two reversible adiabatics (Fig. 13.13(d)). While in Rankine cycle, the working fluid undergoes phase change, in Brayton cycle the working fluid always remains in the gaseous phase. Both the pump and the steam turbine in the case of Rankine cycle, and the compressor and the gas turbine in the case of Brayton cycle operate through the same pressure difference of p_1 and p_2. All are steady-flow machines and the work transfer is given by $-\int_{p_1}^{p_2} v\, dp$. For Brayton cycle, the average specific volume of air handled by the compressor is less than the same of gas in the gas turbine (since the gas temperature is much higher), the work done by the gas turbine is more than the work input to the compressor, so that there is W_{net} available to deliver. In the case of Rankine cycle, the specific volume of water in the pump is much less than that of the steam expanding in the steam turbine, so $W_T >> W_P$. Therefore, steam power plants are more popular than the gas turbine plants for electricity generation.

13.9.1 Comparison between Brayton Cycle and Otto Cycle

Brayton and Otto cycles are shown superimposed on the $p - v$ and $T - s$ diagrams in Fig. 13.14. For the same r_k and work capacity, the Brayton cycle (1-2-5-6) handles a larger range of volume and a smaller range of pressure and temperature than does the Otto cycle (1-2-3-4).

In the reciprocating engine field, the Brayton cycle is not suitable. A reciprocating engine cannot efficiently handle a large volume flow of low pressure gas, for which the engine size ($\pi/4D^2 L$) becomes large, and the friction losses also become more. So the Otto cycle is more suitable in the reciprocating engine field.

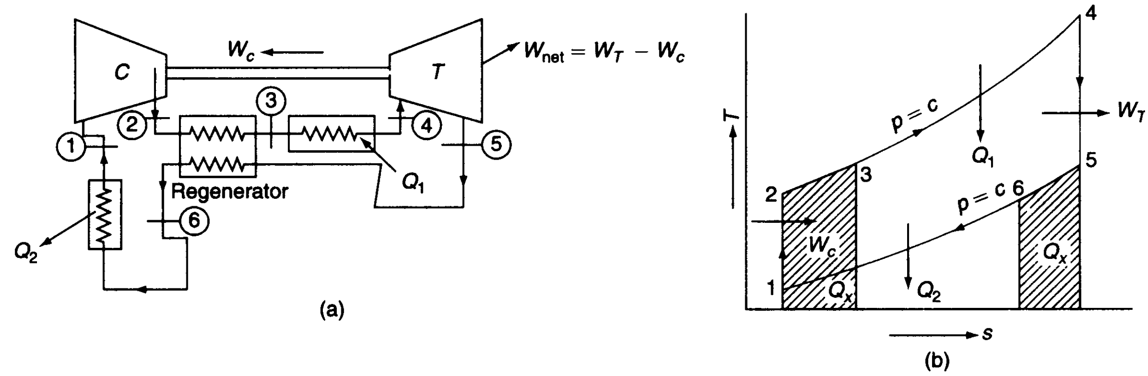


Comparison of otto and brayton cycles

In turbine plants, however, the Brayton cycle is more suitable than the Otto cycle. An internal combustion engine is exposed to the highest temperature (after the combustion of fuel) only for a short while, and it gets time to become cool in the other processes of the cycle. On the other hand, a gas turbine plant, a steady flow device, is always exposed to the highest temperature used. So to protect material, the maximum temperature of gas that can be used in a gas turbine plant cannot be as high as in an internal combustion engine. Also, in the steady flow machinery, it is more difficult to carry out heat transfer at constant volume than at constant pressure. Moreover, a gas turbine can handle a large volume flow of gas quite efficiently. So we find that the Brayton cycle is the basic air standard cycle for all modern gas turbine plants.

13.9.2 Effect of Regeneration on Brayton Cycle Efficiency

The efficiency of the Brayton cycle can be increased by utilizing part of the energy of the exhaust gas from the turbine in heating up the air leaving the compressor in a heat exchanger called a *regenerator*, thereby reducing the amount of heat supplied from an external source and also the amount of heat rejected. Such a cycle is illustrated in Fig. 13.15. The temperature of air leaving the turbine at 5 is higher than that of air leaving the compressor at 2. In the regenerator, the temperature of air leaving the compressor is raised by heat transfer from the turbine exhaust. The maximum temperature to which the cold air at 2 could be heated is the temperature of the hot air leaving the turbine at 5. This is possible only in an infinite heat exchanger. In the real case, the temperature at 3 is less than that at 5. The ratio of the actual temperature rise of air to the maximum possible rise is called the *effectiveness* of the regenerator. For this case illustrated



Effect of regeneration on Brayton cycle

$$\text{Effectiveness} = \frac{t_3 - t_2}{t_5 - t_2}$$

When the regenerator is used in the idealized cycle (Fig. 13.15), the heat supplied and the heat rejected are each reduced by the same amount, Q_x. *The mean temperature of heat addition increases and the mean temperature of heat rejection decreases because of the use of the regenerator.* The efficiency is increased as a result, but the work output of the cycle remains unchanged. Here,

$$Q_1 = h_4 - h_3 = c_p (T_4 - T_3) \qquad Q_2 = h_6 - h_1 = c_p (T_6 - T_1)$$

$$W_T = h_4 - h_5 = c_p (T_4 - T_5) \qquad W_c = h_2 - h_1 = c_p (T_2 - T_1)$$

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_6 - T_1}{T_4 - T_3}$$

In practice the regenerator is costly, heavy and bulky, and causes pressure losses which bring about a decrease in cycle efficiency. These factors have to be balanced against the gain in efficiency to decide whether it is worthwhile to use the regenerator.

Above a certain pressure ratio (p_2/p_1) the addition of a regenerator causes a loss in cycle efficiency when compared to the original Brayton cycle. In this situation the compressor discharge temperature (T_2) is higher than the turbine exhaust gas temperature (T_5) (Fig. 13.18). The compressed air will thus be cooled in the regenerator and the exhaust gas will be heated. As a result both the heat supply and heat rejected are increased. However, the compressor and turbine works remain unchanged. So, the cycle efficiency (W_{net}/Q_1) decreases.

Let us now derive an expression for the ideal regenerative cycle when the compressed air is heated to the turbine exhaust temperature in the regenerator so that $T_3 = T_5$ and $T_2 = T_6$ (Fig. 13.15). Therefore,

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_6 - T_1}{T_4 - T_3} = 1 - \frac{T_1}{T_4}\left[\frac{(T_2/T_1) - 1}{1 - (T_5/T_4)}\right] = 1 - \frac{T_1}{T_4} \cdot \frac{T_2}{T_1}\left[\frac{1 - (T_1/T_2)}{1 - (T_5/T_4)}\right]$$

Since $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma} = \frac{T_4}{T_5} \qquad \eta = 1 - \frac{T_1}{T_4} r_p^{\gamma-1/\gamma} \qquad (13.17)$$

For a fixed ratio of (T_1/T_4), the cycle efficiency drops with increasing pressure ratio.

13.9.3 Effect of Irreversibilities in Turbine and Compressor

The Brayton cycle is highly sensitive to the real machine efficiencies of the turbine and the compressor. Figure 13.16 shows the actual and ideal expansion and compression processes.

Turbine efficiency, $$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

Compressor efficiency, $$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$W_{net} = W_T - W_C = (h_3 - h_4) - (h_2 - h_1)$$

$$Q_1 = h_3 - h_2 \text{ and } Q_2 = h_4 - h_1$$

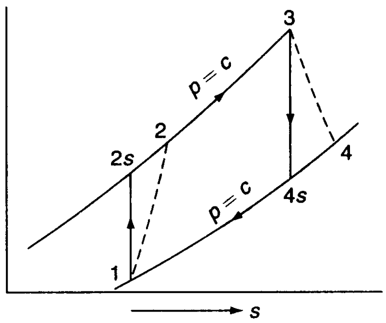


Fig. 13.16 *Effect of machine efficiencies on Brayton cycle*

The net output is reduced by the amount $[(h_4 - h_{4s}) + (h_2 - h_{2s})]$, and the heat supplied is reduced by the amount $(h_2 - h_{2s})$.

The efficiency of the cycle will thus be less than that of the ideal cycle. As η_T and η_C decrease, η_{cycle} also decreases. The cycle efficiency may approach zero even when η_T and η_C are of the order of 60 to 70%. This is the main drawback of a gas turbine plant. The machines have to be highly efficient to obtain reasonable cycle efficiency.

13.9.4 Effect of Pressure Ratio on the Brayton Cycle

The efficiency of the Brayton cycle is a function of the pressure ratio as given by the equation

$$\eta = 1 - \frac{1}{(r_p)^{(\gamma-1)/\gamma}}$$

The more the pressure ratio, the more will be the efficiency.

Let T_1 = the lowest temperature of the cycle, which is the temperature of the surroundings (T_{min}), and T_3 = the maximum or the highest temperature of the cycle limited by the characteristics of the material available for burner and turbine construction (T_{max}).

Since the turbine, a steady-flow machine, is always exposed to the highest temperature gas, the maximum temperature of gas at the inlet to the turbine is limited to about 800°C by using a high air-fuel ratio. With turbine blade cooling, however, the maximum gas inlet temperature can be 1100°C or even higher.

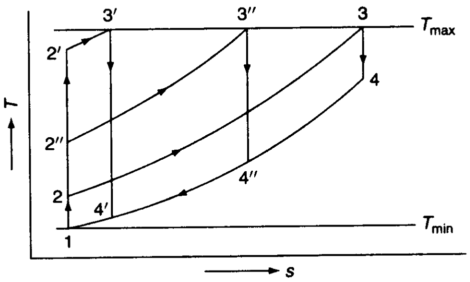


Fig. 13.17 Effect of pressure ratio on brayton cycle

Figure 13.17 shows the Brayton cycles operating between the same T_{max} and T_{min} at various pressure ratios. As the pressure ratio changes, the cycle shape also changes. For the cycle 1-2-3-4 of low pressure ratio r_p, since the average temperature of heat addition

$$T_{m1} = \frac{h_3 - h_2}{s_3 - s_2}$$

is only a little greater than the average temperature of heat rejection

$$T_{m2} = \frac{h_4 - h_1}{s_4 - s_1}$$

the efficiency will be low. *At the lower limit of unity pressure ratio, both work output and efficiency will be zero.*

As the pressure ratio is increased, the efficiency steadily increases, because T_{m1} increases and T_{m2} decreases. The mean temperature of heat addition T_{m1} approaches T_{max} and the mean temperature of heat rejection T_{m2} approaches T_{min}, with the increase in r_p. *In the limit when the compression process ends at T_{max}, the Carnot efficiency is reached, r_p has the maximum value (r_{pmax}), but the work capacity again becomes zero.*

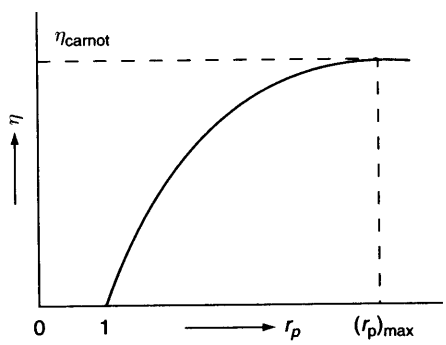


Fig. 13.18 Effect of pressure ratio on brayton cycle efficiency

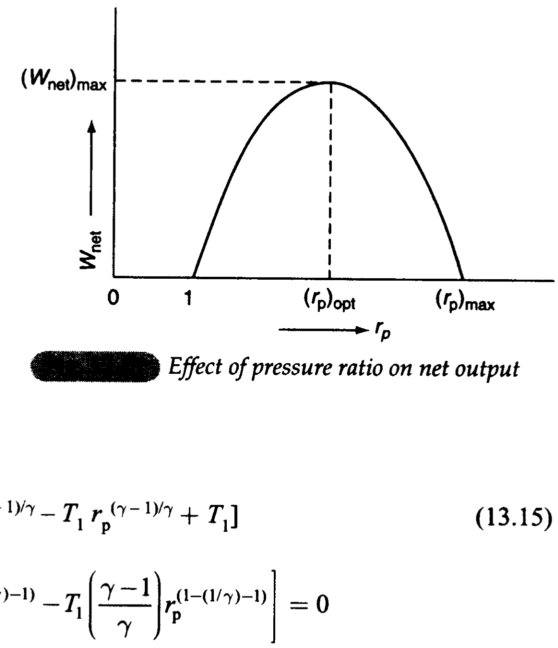
Figure 13.18 shows how the cycle efficiency varies with the pressure ratio, with r_p varying between two limiting values of 1 and $(r_p)_{max}$ when the Carnot efficiency is reached. When r_p = $(r_p)_{max}$, the cycle efficiency is given by

$$\eta = 1 - \frac{1}{((r_p)_{max})^{\gamma-1/\gamma}} = \eta_{Carnot} = 1 - \frac{T_{min}}{T_{max}}$$

$$\therefore \quad ((r_p)_{max})^{(\gamma-1)/\gamma} = \frac{T_{max}}{T_{min}}$$

$$\therefore \quad (r_p)_{max} = \left(\frac{T_{max}}{T_{min}}\right)^{\gamma/(\gamma-1)} \tag{13.14}$$

From Fig. 13.17 it is seen that the work capacity of the cycle, operating between T_{max} and T_{min}, is zero when $r_p = 1$ passes through a maximum, and then again becomes zero when the Carnot efficiency is reached. There is an optimum value of pressure ratio $(r_p)_{opt}$ at which work capacity becomes a maximum, as shown in Fig. 13.19.



Effect of pressure ratio on net output

For 1 kg,

$$W_{net} = c_p\,[(T_3 - T_4) - (T_2 - T_1)]$$

where $T_3 = T_{max}$ and $T_1 = T_{min}$

Now $\dfrac{T_3}{T_4} = (r_p)^{(\gamma-1)/\gamma}$

$$\therefore \quad T_4 = T_3 \cdot r_p^{-(\gamma-1)/\gamma}$$

Similarly $T_2 = T_1 \cdot r_p^{(\gamma-1)/\gamma}$

Substituting in the expression for W_{net}

$$W_{net} = c_p\,[T_3 - T_3 \cdot (r_p)^{-(\gamma-1)/\gamma} - T_1\, r_p^{(\gamma-1)/\gamma} + T_1] \tag{13.15}$$

To find $(r_p)_{opt}$

$$\frac{dW_{net}}{dr_p} = c_p\left[-T_3\left(-\frac{\gamma-1}{\gamma}\right) r_p^{(-1+(1/\gamma)-1)} - T_1\left(\frac{\gamma-1}{\gamma}\right) r_p^{(1-(1/\gamma)-1)}\right] = 0$$

$$\therefore \quad T_3\left(\frac{\gamma-1}{\gamma}\right) r_p^{(1/\gamma)-2} = T_1\left(\frac{\gamma-1}{\gamma}\right) \cdot r_p^{-1/\gamma}$$

$$\therefore \quad r_p^{-(1/\gamma)-(1/\gamma)+2} = \frac{T_3}{T_1}$$

or

$$(r_p)_{opt} = \left(\frac{T_3}{T_1}\right)^{\gamma/2(\gamma-1)}$$

$$\therefore \quad (r_p)_{opt} = \left(\frac{T_{max}}{T_{min}}\right)^{\gamma/2(\gamma-1)} \tag{13.16}$$

From Eqs. (13.14) and (13.26)

$$(r_p)_{opt} = \sqrt{(r_p)_{max}} \tag{13.17}$$

Substituting the values of $(r_p)_{opt}$ in Eq. (13.15)

$$W_{net} = (W_{net})_{max} = c_p\left[T_3 - T_3\left(\frac{T_1}{T_3}\right)^{\frac{\gamma}{2(\gamma-1)}\cdot\frac{\gamma-1}{\gamma}} - T_1\left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}\cdot\frac{\gamma-1}{\gamma}} + T_1\right] = c_p[T_3 - 2\sqrt{T_1T_3} + T_1]$$

or
$$(W_{net})_{max} = c_p(\sqrt{T_{max}} - \sqrt{T_{min}})^2 \tag{13.18}$$

$$\eta_{cycle} = 1 - \frac{1}{r_p^{\gamma-1/\gamma}} = 1 - \sqrt{\frac{T_{min}}{T_{max}}}\,. \tag{13.19}$$

Considering the cycles 1-2′-3′-4′ and 1-2″-3″-4″ (Fig. 13.17), it is obvious that to obtain a reasonable work capacity, a certain reduction in efficiency must be accepted.

13.9.5 Effect of Intercooling and Reheating on Brayton Cycle

The efficiency of the Brayton cycle may often be increased by the use of staged compression with intercooling, or by using staged heat supply, called reheat.

Let the compression process be divided into two stages. Air, after being compressed in the first stage, is cooled to the initial temperature in a heat exchanger, called an intercooler, and then compressed further in the second stage (Fig. 13.20). 1-2′-5-6 is the ideal cycle without intercooling, having a single-stage compression, 1-2-3-4-6 is the cycle with intercooling, having a two-stage compression. The cycle 2-3-4-2′ is thus added to the basic cycle 1-2′-5-6. There is more work capacity, since the included area is more. There is more heat supply also. For the cycle 4-2′-2-3, T_{m1} is lower and T_{m2} higher (lower r_p) than those of the basic cycle 1-2′-5-6. So the efficiency of the cycle reduces by staging the compression and intercooling. But if a regenerator is used, the low temperature heat addition (4–2′) may be obtained by recovering energy from the exhaust gases from the turbine. So there may be a net gain in efficiency when intercooling is adopted in conjunction with a regenerator.

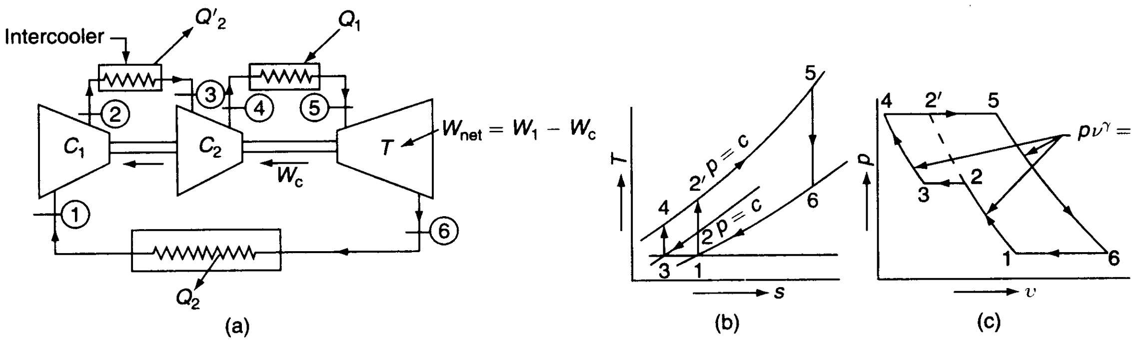
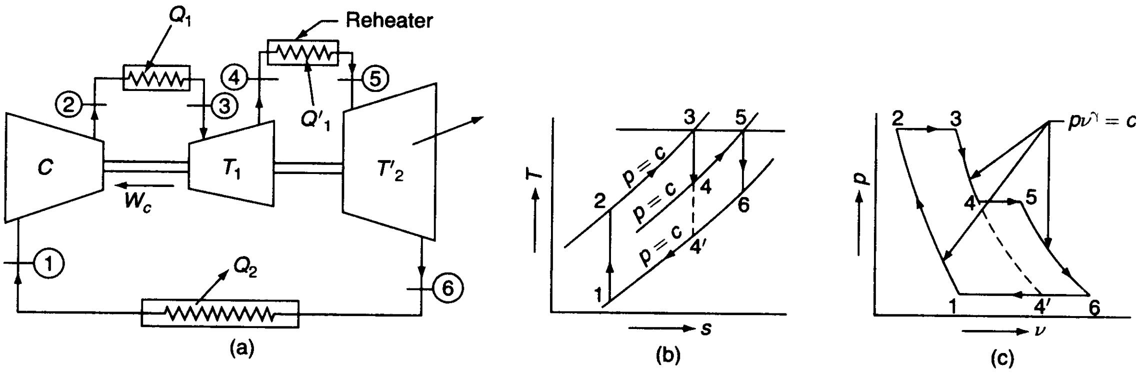


Fig. 13.20 *Effect of intercooling on Brayton cycle*

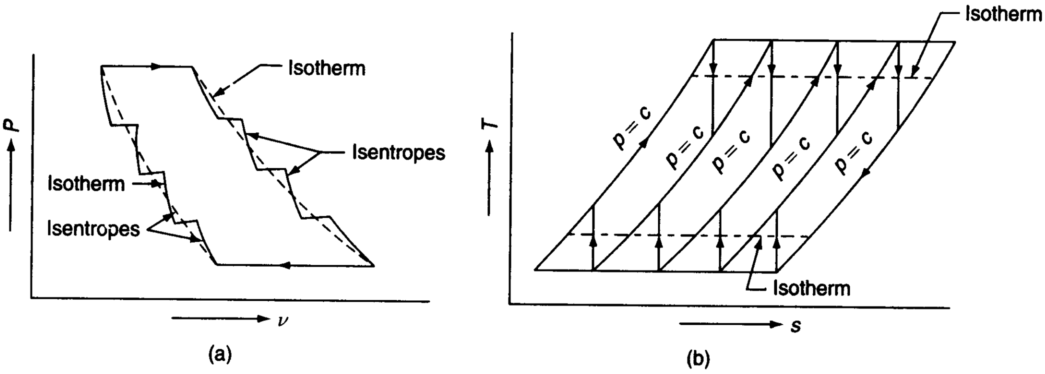
Similarly, let the total heat supply be given in two stages and the expansion process be divided in stages in two turbines (T_1 and T_2) with intermediate reheat, as shown in Fig. 13.21. 1-2-3-4′ is the cycle with a single-stage heat supply having no reheat, with total expansion in one turbine only. 1-2-3-4-5-6 is the cycle with a single-stage reheat, having the expansion divided into two stages. With the basic cycle, the cycle 4-5-6-4 is added because of reheat. The work capacity increases, but the heat supply also increases.

In the cycle 4-5-6-4′, r_p is lower than in the basic cycle 1-2-3-4′, so its efficiency is lower. Therefore, the efficiency of the cycle decreases with the use of reheat. But T_6 is greater than T'_4. Therefore, if regeneration is employed, there is more energy that can be recovered from the turbine exhaust gases. So when regeneration is employed in conjunction with reheat, there may be a net gain in cycle efficiency.

If in one cycle, several stages of intercooling and several stages of reheat are employed, a cycle as shown in Fig. 13.22 is obtained. When the number of such stages is large the cycle reduces to the Ericsson cycle with two reversible isobars and two reversible isotherms. With ideal regeneration the cycle efficiency becomes equal to the Carnot efficiency.



Effect of reheat on Brayton cycle



Brayton cycle with many stages of intercooling and reheating approximates to ericsson cycle

13.9.6 Ideal Regenerative Cycle with Intercooling and Reheat

Let us consider an ideal regenerative gas turbine cycle with two-stage compression and a single reheat. It assumes that both intercooling and reheating take place at the root mean square of the high and low pressure in the cycle, so that $p_3 = p_2 = p_7 = p_8 = \sqrt{p_1 p_4} = \sqrt{p_6 p_9}$ (Fig. 13.23). Also, the temperature after intercooling is equal to the compressor inlet temperature ($T_1 = T_3$) and the temperature after reheat is equal to the temperature entering the turbine initially ($T_6 = T_8$).

Here, $$Q_1 = c_p(T_6 - T_5) + c_p(T_8 - T_7)$$

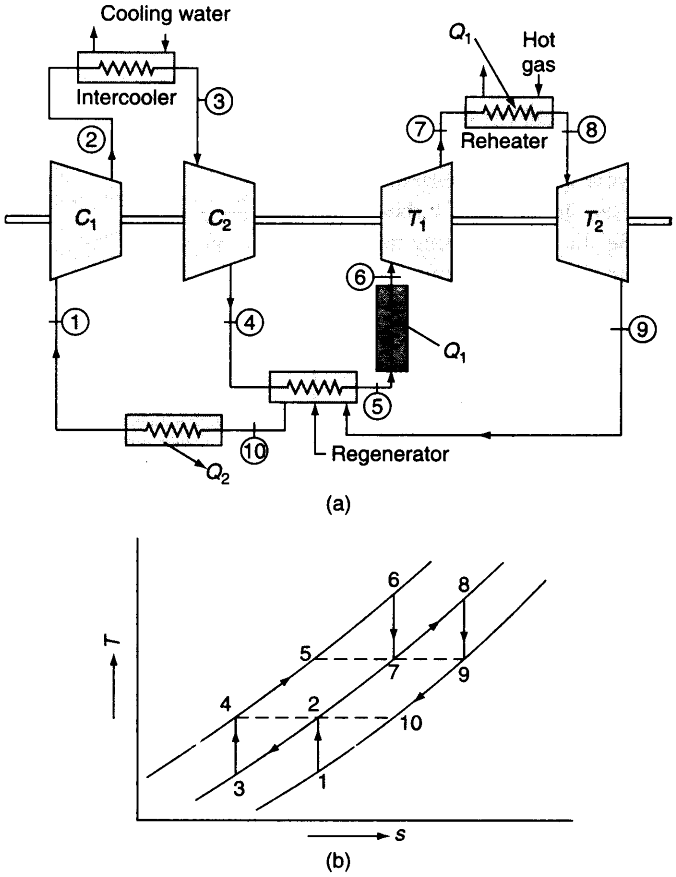
Since $\frac{p_6}{p_7} = \frac{p_8}{p_9}$ and $T_6 = T_8$, it follows that $T_5 = T_7 = T_9$.

$\therefore$ $$Q_1 = 2c_p(T_6 - T_7)$$

Again, $$Q_2 = c_p(T_{10} - T_1) + c_p(T_2 - T_3)$$

but $$\frac{p_2}{p_1} = \frac{p_4}{p_3} \quad \text{and} \quad T_3 = T_1$$

so that $$T_2 = T_4 = T_{10}$$



[illegible] *Ideal regenerative gas turbine cycle with two-stage compression and reheat*

$$\therefore \quad Q_2 = 2c_p(T_2 - T_1)$$

$$W_{net} = Q_1 - Q_2 = 2c_p[(T_6 - T_7) - (T_2 - T_1)]$$

$$\therefore \eta_{cycle} = \frac{W_{net}}{Q_1} = 1 - \frac{T_2 - T_1}{T_6 - T_7} = 1 - \frac{T_1}{T_6}\left[\frac{(T_2/T_1) - 1}{1 - (T_7/T_6)}\right] = 1 - \frac{T_1}{T_6} \times \frac{T_2}{T_1}\left[\frac{1 - (T_1/T_2)}{1 - (T_7/T_6)}\right] = 1 - \frac{T_1}{T_6} \times \left(\frac{p_2}{p_1}\right)^{\gamma - 1/\gamma}$$

But $$p_2 = \sqrt{p_1 p_4},$$

$$\therefore \quad \eta_{cycle} = 1 - \frac{T_1}{T_6}\left(\frac{p_4}{p_1}\right)^{(\gamma-1)/2\gamma} \tag{13.20}$$

13.9.7 Free-shaft Turbine

So far only a single shaft has been shown in flow diagrams, on which were mounted all the compressors and the turbines. Sometimes, for operating convenience and part-load efficiency, one turbine is used for driving the compressor only on one shaft, and a separate turbine is used on another shaft, known as free-shaft, for supplying the load, as shown in Fig. 13.24.

13.10 AIRCRAFT PROPULSION

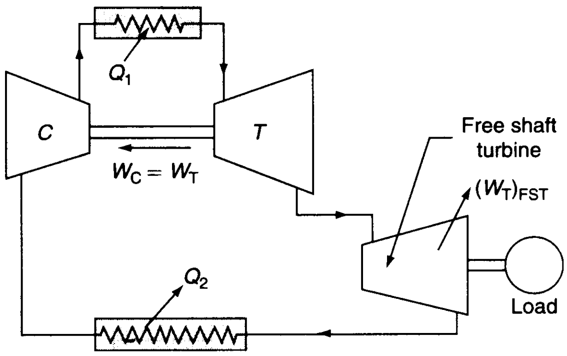


Fig. 13.24 *Free shaft turbine*

Gas turbines are particularly suited for aircraft propulsion because they are light and compact and have a high power-to-weight ratio. Aircraft gas turbines operate on an open cycle called *"jet propulsion cycle"*, as explained below. It can be of turbojet, turbofan or turborop type. In a turbojet engine (Fig. 13.25), high velocity air first flows through a diffuser where it is decelerated increasing its pressure. Air is then compressed in the compressor. It is mixed with fuel in the combustion chamber, where the mixture is burned at constant pressure. The high pressure, high temperature combustion gases partially expand in the turbine, producing enough power to drive the compressor and any auxiliary equipment. Finally, the gases expand in the nozzle to the ambient pressure and leave the aircraft at high velocity. In the ideal case, the turbine work is assumed to be equal to the compressor work. The processes in the diffuser, the compressor, the turbine, and the nozzle are assumed to be reversible and adiabatic.

The thrust developed in a turbojet engine is the unbalanced force caused by the difference in the momentum of the air entering the engine and the exhaust gases leaving the engine, so that,

$$F = (\dot{m}\bar{V})_{exit} - (\dot{m}\bar{V})_{inlet} = \dot{m}(\bar{V}_{exit} - \bar{V}_{inlet}) \tag{13.21}$$

The pressures at inlet and exit of the engine are the ambient pressure. For an aircraft cruising in still air, $\bar{V}$ inlet is the aircraft velocity. The mass flow rates of the gases at the engine exit and the inlet are different, the difference being equal to the combustion rate of the fuel. But the air-fuel ratio used in jet propulsion engines is usually very high, making the difference very small. Thus $\dot{m}$ in Eq. (13.21) is taken as the mass flow rate of air through the engine. For an aircraft cruising at a steady speed, the thrust is used to overcome the fluid drag, and the net force acting on the body of the aircraft is zero. Commercial airplanes save fuel by flying at higher altitudes during long trips since the air at higher altitudes is of less density and exerts a smaller drag force on the aircraft.

The power developed from the thrust of the engine is called the propulsive power, W_p, given by:

$$\dot{W}_p = F\bar{V}_{aircraft} = \dot{m}(\bar{V}_{exit} - \bar{V}_{inlet})\bar{V}_{aircraft} \tag{13.22}$$

The propulsive efficiency, η_p, is defined by:

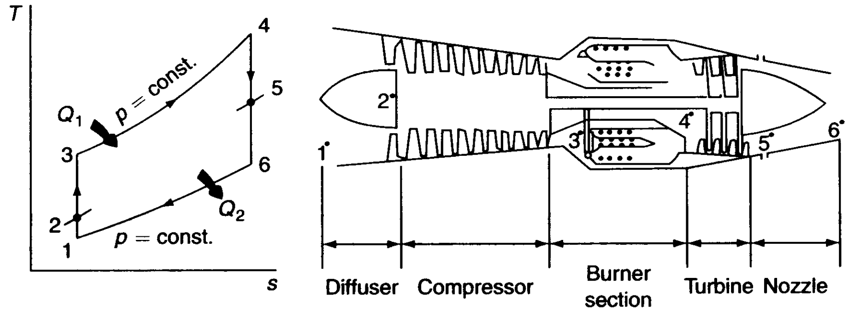


Fig. 13.25 *Basic components of a turbojet engine and the T–s diagram of an ideal turbojet cycle*

$$\eta_p = \frac{\text{Propulsive power}}{\text{Energy input rate}} = \frac{\dot{W}_P}{\dot{Q}_{in}} \tag{13.23}$$

It is a measure of how efficiently the energy released during combustion is converted to propulsive power.

Space and weight limitations prohibit the use of regenerators and intercoolers on aircraft engines. The counterpart of reheating is afterburning. The air-fuel ratio in a jet engine is so high that the turbine exhaust gases are sufficiently rich in oxygen to support the combustion of more fuel in an afterburner (Fig. 13.26). Such burning of fuel raises the temperature of the gas before it expands in the nozzle, increasing the K.E. change in the nozzle and consequently increasing the thrust. In the air-standard case, the combustion is replaced by constant pressure heat addition.

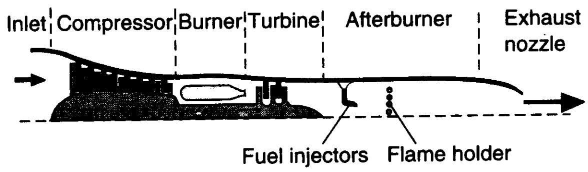


Fig. 13.26 *Turbojet engine with afterburner*

The most widely used engine in aircraft propulsion is the *turbofan engine* wherein a large fan driven by the turbine forces a considerable amount of air through a duct (cowl) surrounding the engine (Figs. 13.27 and 13.28). The fan exhaust leaves the duct at a higher velocity, enhancing the total thrust of the engine significantly. Some of the air entering the engine flows through the compressor, combustion chamber and turbine, and the rest passes through the fan into a duct and is either mixed with the exhaust gases or is discharged separately. It improves the engine performance over a broad operating range. The ratio of the mass flow rates of the two streams is called the *bypass ratio.*

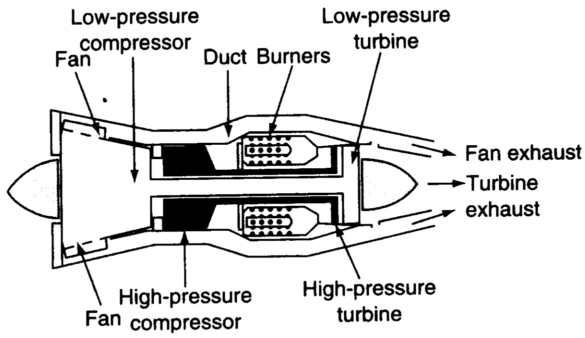


Fig. 13.27 *Turbofan engine*

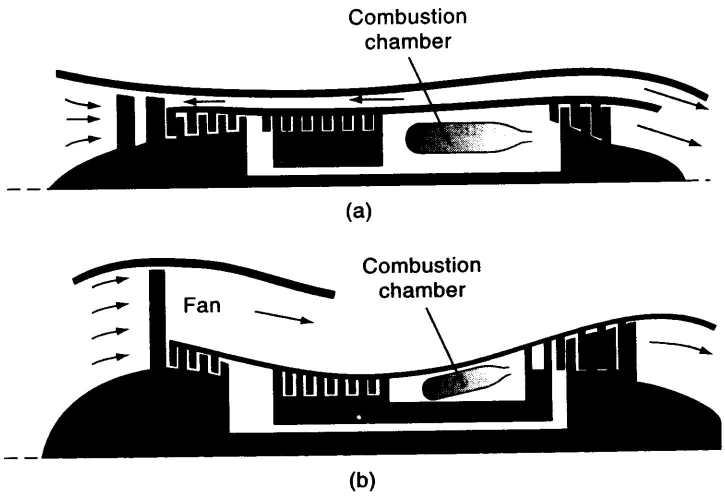


Fig. 13.28 *Turbofan or bypass jet engines*

$$\text{Bypass ratio} = \frac{\dot{m}_{\text{total}} - \dot{m}_{\text{turbine}}}{\dot{m}_{\text{turbine}}}$$

The bypass ratio can be varied in flight by various means. Turbofan engines deserve most of the credit for the success of jumbo jets, which weigh almost 400,000 kg and are capable of carrying 400 passengers for up to 10,000 km at speeds over 950 km/h with less fuel per passenger mile.

Increasing the bypass ratio of a turbofan engine increases thrust. If the cowl is removed from the fan the result is a *turboprop engine* (Fig. 13.29). Turbofan and turboprop engines differ mainly in their bypass ratio 5 or 6 for turbofans and as high as 100 for turboprop. In general, propellers are more efficient than jet engines, but they are limited to low-speed and low-altitude operation since their efficiency decreases at high speeds and altitudes.

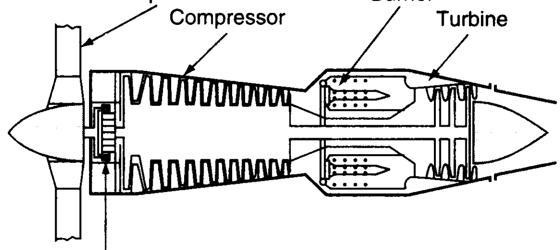


Fig. 13.29 Turboprop engine

A particularly simple type of engine known as a *ramjet* is shown in Fig. 13.30. This engine requires neither a compressor nor a turbine. A sufficient pressure rise is obtained by decelerating the high speed incoming air in the diffuser (ram effect) on being rammed against a barrier. For the ramjet to operate, the aircraft must already be in flight at a sufficiently great speed. The combustion products exiting the combustor are expanded through a nozzle to produce the thrust.

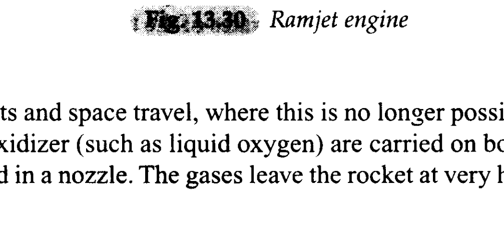


Fig. 13.30 Ramjet engine

In each of the engines mentioned so far, combustion of the fuel is supported by air brought into the engines from the atmosphere. For very high altitude flights and space travel, where this is no longer possible, rockets may be employed. In a rocket, both fuel and an oxidizer (such as liquid oxygen) are carried on board of the craft. High pressure combustion gases are expanded in a nozzle. The gases leave the rocket at very high velocities, producing the thrust to propel the rocket.

13.11 BRAYTON-RANKINE COMBINED CYCLE

Both Rankine cycle and Brayton cycle consist of two reversible isobars and two reversible adiabatics. While the former is a phase change cycle, in the latter the working fluid does not undergo any phase change.

A gas turbine power plant operating on Brayton cycle has certain disadvantages like large compressor work, large exhaust loss, sensitivity to machine inefficiencies (η_T and η_C), relatively lower cycle efficiency and costly fuel. Due to these factors, the cost of power generation by a stationary gas turbine in a utility system is high. However, a gas turbine plant offers certain advantages also, such as less installation cost, less installation time, quick starting and stopping, and fast response to load changes. So, a gas turbine plant is often used as a *peaking unit* for certain hours of the day, when the energy demand is high. To utilize the high temperature exhaust and to raise its plant efficiency a gas turbine may be used in conjunction with a steam turbine plant to offer the gas turbine advantages of quick starting and stopping and permit flexible operation of the combined plant over a wide range of loads.

Let us consider two cyclic power plants coupled in series, the topping plant operating on Brayton cycle and the bottoming one operating on Rankine cycle (Fig. 13.31). Helium may be the working fluid in the topping plant and water in the bottoming plant. The overall efficiency of the combined plant is:

$$\eta = \eta_1 + \eta_2 - \eta_1 \eta_2$$

where η_1 and η_2 are the efficiencies of the Brayton cycle and Rankine cycle respectively.

For capacity augmentation often supplementary firing is used (Fig. 13.32). For expansion of combustion gases in the gas turbine

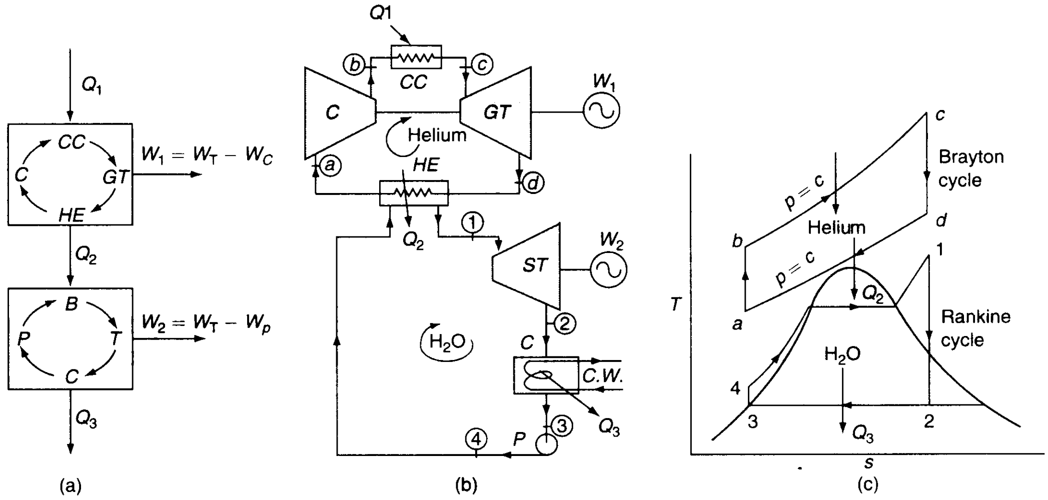


Fig. 13.31 *Brayton-Rankine combined cycle plant*

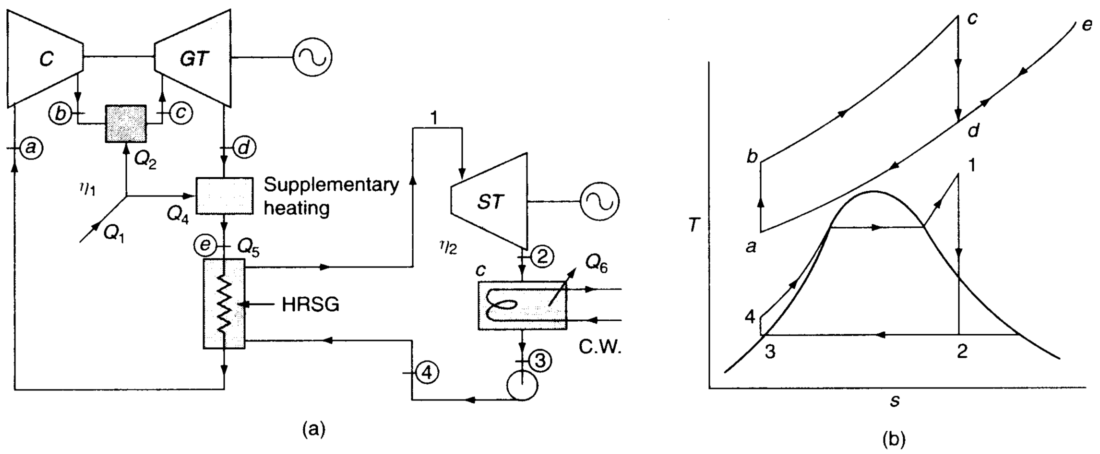


Fig. 13.32 *Brayton/Rankine cyclic plants with supplementary heating*

$$T_c/T_d = (p_2/p_1)^{(\gamma - 1)/\gamma}$$

where $\gamma = 1.3$ (assumed).

For the compressor, $$T_b/T_a = (p_2/p_1)^{(\gamma' - 1)/\gamma'}$$

where $$\gamma' = 1.4.$$

$$W_{GT} = w_a [c_{pg} (T_c - T_d) - c_{pa} (T_b - T_a)]$$

neglecting the mass of fuel (for a high air-fuel ratio), and w_a being the mass flow of air.

$$W_{ST} = w_s (h_1 - h_2)$$

where w_s is the steam flow rate. The pump work is neglected. By energy balance,

$$w_a c_{pg} (T_c - T_f) = w_s (h_1 - h_4)$$

Now, $$Q_1 = w_a c_{pg} [(T_c - T_b) + (T_e - T_d)]$$

The overall efficiency of the plant is:

$$\eta = \frac{W_{GT} + W_{ST}}{Q_1}$$

Again, $$Q_1 = w_f \times \text{C.V.}$$

where w_f is the fuel burning rate.

High overall efficiency, low investment cost, less water requirement, large operating flexibility, phased installation, and low environmental impact are some of the advantages of combined gas-steam cycles.

Solved Examples

Example 13.1

An engine working on the Otto cycle is supplied with air at 0.1 *MPa,* 35°*C. The compression ratio is* 8. *Heat supplied is* 2100 *kJ/kg. Calculate the maximum pressure and temperature of the cycle, the cycle efficiency, and the mean effective pressure. (For air,* $c_p = 1.005$, $c_v = 0.718$, *and* $R = 0.287$ *kJ/kg K).*

Solution From Fig. Ex. 13.1

$$T_1 = 273 + 35 = 308 \text{ K}$$

$$p_1 = 0.1 \text{ MPa} = 100 \text{ kN/m}^2$$

$$Q_1 = 2100 \text{ kJ/kg}$$

$$r_k = 8, \gamma = 1.4$$

$$\therefore \quad \eta_{cycle} = 1 - \frac{1}{r_k^{\gamma-1}} = 1 - \frac{1}{8^{0.4}} = 1 - \frac{1}{2.3} = 0.565 \text{ or } 56.5\% \quad \textit{Ans.}$$

$$\frac{v_1}{v_2} = 8, \; v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 308}{100} = 0.884 \text{ m}^3\text{/kg}$$

$$\therefore \quad v_2 = \frac{0.884}{8} = 0.11 \text{ m}^3\text{/kg}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (8)^{0.4} = 2.3$$

$\therefore \quad T_2 = 2.3 \times 308 = 708.4 \text{ K}$

$Q_1 = c_v (T_3 - T_2) = 2100 \text{ kJ/kg}$

$\therefore \quad T_3 - 708.4 = \dfrac{2100}{0.718} = 2925 \text{ K}$

or $\quad T_3 = T_{max} = 3633 \text{ K}$ *Ans.*

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = (8)^{1.4} = 18.37$$

$\therefore \quad p_2 = 1.873 \text{ MPa}$

Again

$$\frac{p_3 v_3}{T_3} = \frac{p_2 v_2}{T_2}$$

$\therefore \quad p_3 = p_{max} = 1.837 \times \dfrac{3633}{708} = 9.426 \text{ MPa}$ *Ans.*

$W_{net} = Q_1 \times \eta_{cycle} = 2100 \times 0.565$

$= 1186.5 \text{ kJ/kg}$

$= p_m(v_1 - v_2) = p_m(0.884 - 0.11)$

$\therefore \quad p_m = \text{m.e.p.} = \dfrac{1186.5}{0.774} = 1533 \text{ kPa} = 1.533 \text{ MPa}$ *Ans.*

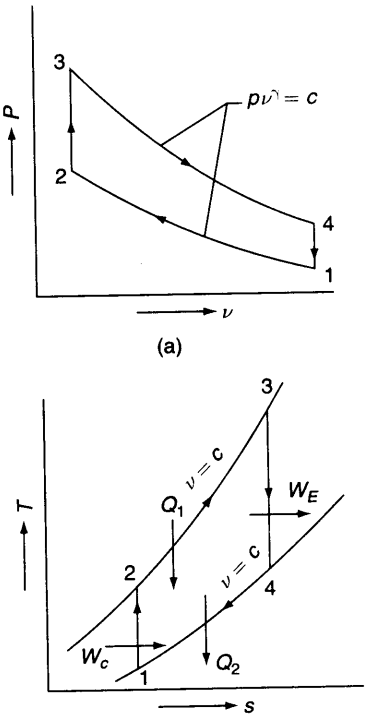


Fig. Ex. 13.1

Example 13.2

A Diesel engine has a compression ratio of 14 and cut-off takes place at 6% of the stroke. Find the air standard efficiency.

Solution From Fig. Ex. 13.2

$$r_k = \frac{v_1}{v_2} = 14$$

$v_3 - v_2 = 0.06\,(1\,v_1 - v_2)$

$= 0.06\,(14\,v_2 - v_2)$

$= 0.78\,v_2$

$v_3 = 1.78\,v_2$

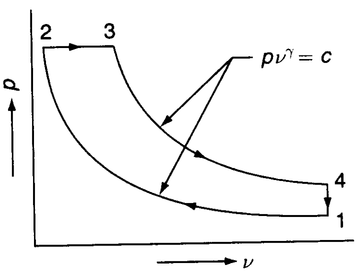


Fig. Ex. 13.2

$\therefore$ Cut-off ratio, $r_c = \frac{v_1}{v_2} = 1.78$

$$\eta_{\text{Diesel}} = 1 - \frac{1}{\gamma} \cdot \frac{1}{r_k^{\gamma-1}} \cdot \frac{r_c^\gamma - 1}{r_c - 1} = 1 - \frac{1}{1.4} \cdot \frac{1}{(14)^{0.4}} \frac{(1.78)^{1.4} - 1}{1.78 - 1}$$

$$= 1 - 0.248 \cdot \frac{1.24}{0.78} = 0.605, \text{ i.e., } 60.5\%$$ *Ans.*

Example 13.3

In an air standard Diesel cycle, the compression ratio is 16, and at the beginning of isentropic compression, the temperature is 15°C and the pressure is 0.1 MPa. Heat is added until the temperature at the end of the constant pressure process is 1480°C. Calculate (a) the cut-off ratio, (b) the heat supplied per kg of air, (c) the cycle efficiency, and (d) the m.e.p.

Solution From Fig. Ex. 13.3

$$r_k = \frac{v_1}{v_2} = 16$$

$$T_1 = 273 + 15 = 288 \text{ K}$$

$$p_1 = 0.1 \text{ MPa} = 100 \text{ kN/m}^2$$

$$T_3 = 1480 + 273 = 1753 \text{ K}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (16)^{0.4} = 3.03$$

$\therefore$ $T_2 = 288 \times 3.03 = 873$ K

$$\frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3}$$

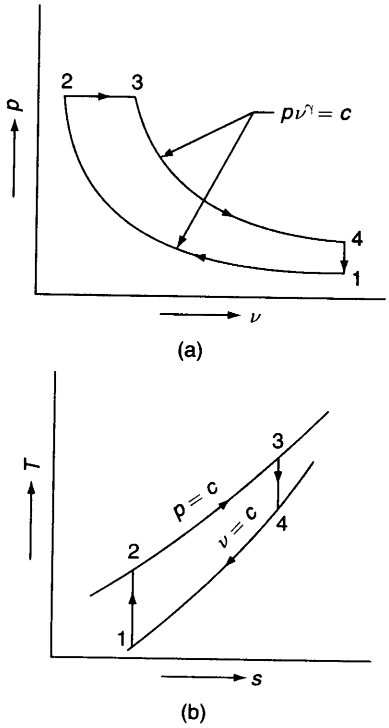


Fig. Ex. 13.3

(a) Cut-off ratio,

$$r_c = \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{1753}{873} = 2.01$$ *Ans.*

(b) Heat supplied, $Q_1 = c_p (T_3 - T_2)$

$$= 1.005 (1753 - 873)$$

$$= 884.4 \text{ kJ/kg}$$

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = \left(\frac{v_1}{v_2} \times \frac{v_2}{v_3}\right)^{\gamma-1} = \left(\frac{16}{2.01}\right)^{0.4} = 2.29$$

$\therefore \quad T_4 = \frac{1753}{2.29} = 766 \text{ K}$

Heat rejected, $Q_2 = c_v (T_4 - T_1) = 0.718 (766 - 288) = 343.2$ kJ/kg

(c) Cycle efficiency $= 1 - \frac{Q_2}{Q_1} = 1 - \frac{343.2}{884.4} = 0.612$ or 61.2% *Ans.*

It may also be estimated from the equation

$$\eta_{\text{cycle}} = 1 - \frac{1}{\gamma} \cdot \frac{1}{r_k^{\gamma-1}} \cdot \frac{r_c^\gamma - 1}{r_c - 1} = 1 - \frac{1}{1.4} \frac{1}{(16)^{0.4}} \cdot \frac{(2.01)^{1.4} - 1}{2.01 - 1}$$

$$= 1 - \frac{1}{1.4} \cdot \frac{1}{3.03} \cdot 1.64 = 0.612 \text{ or } 61.2\%$$ *Ans.*

$$W_{\text{net}} = Q_1 \times \eta_{\text{cycle}} = 884.4 \times 0.612 = 541.3 \text{ kJ/kg}$$

$$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 288}{100} = 0.827 \text{ m}^3\text{/kg}$$

$$v_2 = \frac{0.827}{16} = 0.052 \text{ m}^3\text{/kg}$$

$$\therefore \quad v_1 - v_2 = 0.827 - 0.052 = 0.775 \text{ m}^3\text{/kg}$$

(d) m.e.p. $= \frac{W_{\text{net}}}{v_1 - v_2} = \frac{541.3}{0.775} = 698.45$ kPa *Ans.*

Example 13.4

An air standard dual cycle has a compression ratio of 16, and compression begins at 1 bar, 50°C. The maximum pressure is 70 bar. The heat transferred to air at constant pressure is equal to that at constant volume. Estimate (a) the pressures and temperatures at the cardinal points of the cycle, (b) the cycle efficiency, and (c) the m.e.p. of the cycle, $c_v = 0.718$ kJ/kg K, $c_p = 1.005$ kJ/kg K.

Solution Given: (Fig. Ex. 13.4)

$$T_1 = 273 + 50 = 323 \text{ K}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (16)^{0.4}$$

$$\therefore \quad T_2 = 979 \text{ K}$$

$$p_2 = p_1 \left(\frac{v_1}{v_2}\right)^{\gamma} = 1.0 \times (16)^{1.4} = 48.5 \text{ bar}$$

$$T_3 = T_2 \cdot \frac{p_3}{p_2} = 979 \times \frac{70}{48.5} = 1413 \text{ K}$$

$$Q_{2-3} = c_v (T_3 - T_2) = 0.718 (1413 - 979) = 312 \text{ kJ/kg}$$

Now

$$Q_{2-3} = Q_{3-4} = c_p\,(T_4 - T_3)$$

$$\therefore \quad T_4 = \frac{312}{1.005} + 1413 = 1723 \text{ K}$$

$$\frac{v_4}{v_3} = \frac{T_4}{T_3} = \frac{1723}{1413} = 1.22$$

$$\therefore \quad \frac{v_5}{v_4} = \frac{v_1}{v_2} \times \frac{v_3}{v_4} = \frac{16}{1.22} = 13.1$$

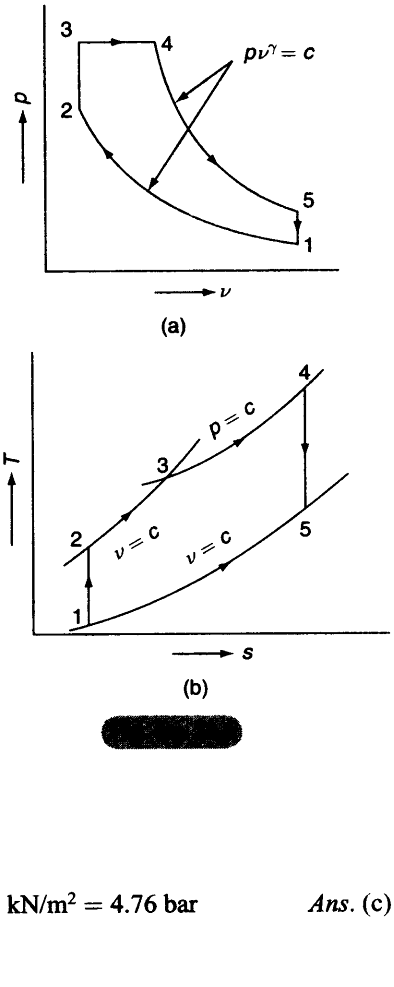
$$\therefore \quad T_5 = T_4 \left(\frac{v_4}{v_5}\right)^{\gamma-1} = 1723 \times \frac{1}{(13.1)^{0.4}} = 615 \text{ K}$$

$$p_5 = p_1 \left(\frac{T_5}{T_1}\right) = 1.0 \times \frac{615}{323} = 1.9 \text{ bar}$$

$$\eta_{\text{cycle}} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_v(T_4 - T_3)}$$

$$= 1 - \frac{0.718(615 - 323)}{312 - 312}$$

$$= 1 - \frac{0.718 \times 292}{624} = 0.665 \text{ or } 66.5\% \qquad \textit{Ans. (b)}$$



[redacted]

$$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \text{ kJ/kg K} \times 323 \text{ K}}{10^2 \text{ kN/m}^2} = 0.927 \text{ m}^3\text{/kg}$$

$$v_1 - v_2 = v_1 - \frac{v_1}{16} = \frac{15}{16} v_1$$

$$W_{\text{net}} = Q_1 \times \eta_{\text{cycle}} = 0.665 \times 624 \text{ kJ/kg}$$

$$\therefore \quad \text{m.e.p.} = \frac{W_{\text{net}}}{v_1 - v_2} = \frac{0.665 \times \text{kJ/kg}}{\frac{15}{16} \times 0.927 \text{ m}^3\text{/kg}} = 476 \text{ kN/m}^2 = 4.76 \text{ bar} \qquad \textit{Ans. (c)}$$

Example 13.5

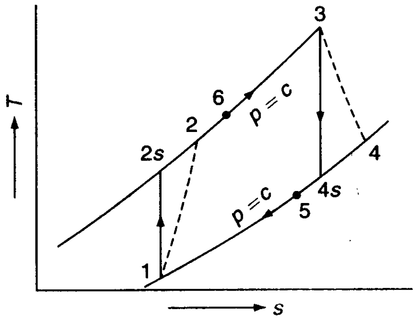
In a gas turbine plant, working on the Brayton cycle with a regenerator of 75% effectiveness, the air at the inlet to the compressor is at 0.1 MPa, 30°C, the pressure ratio is 6, and the maximum cycle temperature is 900°C. If the turbine and compressor have each an efficiency of 80%, find the percentage increase in the cycle efficiency due to regeneration.

Solution Given: (Fig. Ex. 13.5)

$$p_1 = 0.1 \text{ MPa}$$
$$T_1 = 303 \text{ K}$$
$$T_3 = 1173 \text{ K}$$
$$r_p = 6, \eta_T = \eta_C = 0.8$$

Without a regenerator

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = \frac{T_3}{T_{4s}} = (6)^{0.4/1.4} = 1.668$$



$$T_{2s} = 303 \times 1.668 = 505 \text{ K}$$

$$T_{4s} = \frac{1173}{1.668} = 705 \text{ K}$$

$$T_2 - T_1 = \frac{T_{2s} - T_1}{\eta_C} = \frac{505 - 303}{0.8} = 252 \text{ K}$$

$$T_3 - T_4 = \eta_T (T_3 - T_{4s}) = 0.8\,(1173 - 705) = 375 \text{ K}$$

$$W_T = h_3 - h_4 = c_p (T_3 - T_4)$$

$$= 1.005 \times 375 = 376.88 \text{ kJ/kg}$$

$$W_C = h_2 - h_1 = c_p (T_2 - T_1) = 1.005 \times 252 = 253.26 \text{ kJ/kg}$$

$$T_2 = 252 + 303 = 555 \text{ K}$$

$$Q_1 = h_3 - h_2 = c_p (T_3 - T_2) = 1.005\,(1173 - 555) = 621.09 \text{ kJ/kg}$$

$$\therefore \quad \eta = \frac{W_T - W_C}{Q_1} = \frac{376.88 - 253.26}{621.09} = 0.199 \text{ or } 19.9\%$$

With regenerator

$$T_4 = T_3 - 375 = 1173 - 375 = 798 \text{ K}$$

$$\text{Regenerator effectiveness} = \frac{T_6 - T_2}{T_4 - T_2} = 0.75$$

$$\therefore \quad T_6 - 555 = 0.75\,(798 - 555)$$

or $\quad T_6 = 737.3$ K

$$\therefore \quad Q_1 = h_3 - h_6 = c_p (T_3 - T_6) = 1.005\,(1173 - 737.3) = 437.88 \text{ kJ/kg}$$

W_{net} remains the same.

$$\therefore \quad \eta = \frac{W_{net}}{Q_1} = \frac{123.62}{437.9} = 0.2837 \text{ or } 28.37\%$$

$\therefore$ Percentage increase due to regeneration

$$= \frac{0.2837 - 0.199}{0.199} = 0.4256, \text{ or } 42.56\%$$

Example 13.6

A gas turbine plant operates on the Brayton cycle between $T_{min} = 300$ K and $T_{max} = 1073$ K. Find the maximum work done per kg of air, and the corresponding cycle efficiency. How does this efficiency compare with the Carnot cycle efficiency operating between the same two temperatures?

Solution

$$(W_{net})_{max} = c_p \left(\sqrt{T_{max}} - \sqrt{T_{min}}\right)^2 = 1.005\,(\sqrt{1073} - \sqrt{300})^2$$

$$= 1.005\,(15.43)^2 = 239.28 \text{ kJ/kg} \qquad \textit{Ans.}$$

$$\eta_{cycle} = 1 - \frac{1}{(r_p)^{(\gamma-1)/\gamma}} = 1 - \sqrt{\frac{T_{min}}{T_{max}}} = 1 - \sqrt{\frac{300}{1073}} = 0.47 \text{ or } 47\%$$ *Ans.*

$$\eta_{Carnot} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{300}{1073} = 0.721 \text{ or } 72.1\%$$

$$\therefore \quad \frac{\eta_{Brayton}}{\eta_{Carnot}} = \frac{0.47}{0.721} = 0.652$$ *Ans.*

Example 13.7

In an ideal Brayton cycle, air from the atmosphere at 1 atm, 300 K is compressed to 6 atm and the maximum cycle temperature is limited to 1100 K by using a large air-fuel ratio. If the heat supply is 100 MW, find (a) the thermal efficiency of the cycle, (b) work ratio, (c) power output, (d) exergy flow rate of the exhaust gas leaving the turbine.

Solution The cycle efficiency,

$$\eta_{cycle} = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}} = 1 - \frac{1}{6^{0.4/1.4}} = 0.401 \text{ or } 40.1\%$$ *Ans.* (a)

$$T_2/T_1 = (r_p)^{(\gamma-1)/\gamma} = 1.67$$

$$T_2 = 501 \text{ K}$$

$$T_3/T_4 = 1.67, \; T_4 = 1100/1.67 = 658.7 \text{ K}$$

$$W_C = 1.005\,(501 - 300) = 202 \text{ kJ/kg}$$

$$W_T = 1.005\,(1100 - 658.7) = 443.5 \text{ kJ/kg}$$

$$\text{Work ratio} = \frac{W_T - W_C}{W_T} = \frac{241.5}{443.5} = 0.545$$ *Ans.* (b)

$$\text{Power output} = 100 \times 0.401 = 40.1 \text{ MW}$$ *Ans.* (c)

$$Q_1 = \dot{m}\, c_p\,(T_3 - T_2) = 100{,}000 \text{ kW}$$

$$\dot{m} = 166.1 \text{ kg/s}$$

Exergy flow rate of the exhaust gas stream

$$= \dot{m}\, c_p\, T_0 \left(\frac{T_4}{T_0} - 1 - \ln\frac{T_4}{T_0}\right) = 166.1 \times 1.005 \times 300 \left(\frac{658}{300} - 1 - \ln\frac{658.7}{300}\right)$$

$$= 20.53 \text{ MW}$$ *Ans.*

Example 13.8

The following refer to a stationary gas turbine:

Compressor inlet temperature = 311 K

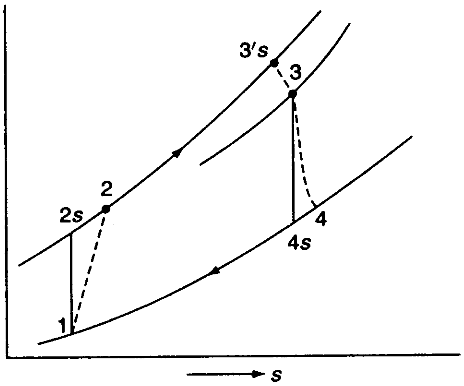
Compressor pressure ratio = 8

Combustion chamber pressure drop = 5% of inlet pressure

Turbine inlet temperature = 1367 *K*
Turbine exit and compressor inlet pressures are atmospheric.

There exists a facility to take air from the compressor exit for use in cooling the turbine. Find the percentage of air that may be taken from the compressor for this purpose so that the overall cycle efficiency drops by 5% *from that of the case of no usage of compressed air for cooling of turbine. For simplicity, assume the following: (a) Take properties of gas through the turbine as those of air, (b) Addition of cooling air to the turbine and addition of fuel to the combustion chamber do not affect the turbine power, (c) Compressor and turbine efficiencies are* 0.87 *and* 0.90 *respectively.*

Solution Given: $\eta_C = 0.87$, $\eta_T = 0.9$, $T_1 = 311$ K, $p_2/p_1 = 8$,
$p_3 = 0.95p_2$, $T_3 = 1367$ K, $p_4 = p_1 = 1$ atm, $\gamma = 1.4$



[redacted]

Case-1: No cooling

$$W_C = \frac{\dot{m}_a c_{pa}(T_{2s} - T_1)}{\eta_c}$$

$$W_T = \dot{m}_g\, c_{pg}\,(T_3 - T_{4s})\,\eta_T$$

$$Q_1 = \dot{m}_c p_c\,(T_3 - T_2)$$

Here, $\dot{m}_a = \dot{m}_g = \dot{m}_c = \dot{m}$

$$c_{pa} = c_{pg} = c_{pc} = c_p$$

$$(T_{2s}/T_1 = p_{2s}/p_1)^{(\gamma-1)/\gamma} = 8^{0.4/1.4} = 1.181$$

$$T_{2s} = 563.3 \text{ K}$$

$$\frac{T_{2s} - T_1}{T_2 - T_1} = 0.87,\ T_2 = 601 \text{ K}$$

$$T_3/T_{4s} = (p_3/p_{4s})^{(\gamma-1)/\gamma} = \left(\frac{0.95p_2}{p_1}\right)^{0.4/1.4} = 1.785$$

$$T_{4s} = 765.83 \text{ K}$$

$$W_C = 290\ \dot{m}c_p,\ W_T = 541.06\ \dot{m}\,c_p \text{ and } Q_1 = 766\ \dot{m}\,c_p$$

$$\eta_{cycle} = \frac{541.06 - 290}{766} = 0.328$$

Case-2: With cooling

$$\eta_{cycle} = 0.328 - 0.05 = 0.278$$

Since the extraction of compressed air for turbine cooling does not contribute to turbine work or burner fuel flow, it can be treated as an increment x added to the compressor mass flow.

$$\frac{541.06 - 290(1+x)}{766} = 0.278$$

$\therefore$ $$x = 0.13$$

% of compressor delivery air flow = $\frac{0.13}{1.13} \times 100 = 11.6\%$ *Ans.*

Example 13.9

In a gas turbine plant the ratio of T_{max}/T_{min} is fixed. Two arrangements of components are to be investigated: (a) single-stage compression followed by expansion in two turbines of equal pressure ratios with reheat to the maximum cycle temperature, and (b) compression in two compressors of equal pressure ratios, with intercooling to the minimum cycle temperature, followed by single-stage expansion. If η_C and η_T are the compressor and turbine efficiencies, show that the optimum specific output is obtained at the same overall pressure ratio for each arrangement.

If η_C is 0.85 and η_T is 0.9, and T_{max}/T_{min} is 3.5, determine the above pressure ratio for optimum specific output and show that with arrangement (a) the optimum output exceeds that of arrangement (b) by about 11%.

Solution (a) With reference to Fig. Ex. 13.9(a)

$$T_1 = T_{min},\ T_3 = T_5 = T_{max},\ \frac{p_2}{p_4} = \frac{p_4}{p_1}$$

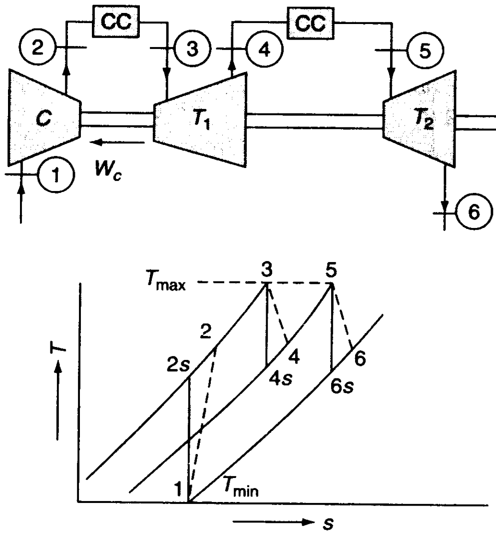
$$\therefore \quad p_4 = \sqrt{p_1 p_2}$$

$$\frac{p_{2s}}{p_1} = r, \text{ pressure ratio}$$

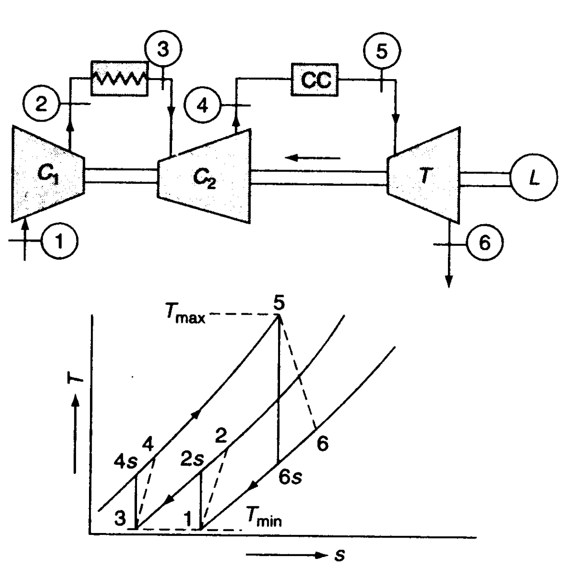
$$\therefore \quad p_{2s} = p_2 = r p_1$$

$$\therefore \quad p_4 = \sqrt{r} \cdot p_1$$

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = r^x$$



(a)



(b)

where $$x = \frac{\gamma - 1}{\gamma}$$

$\therefore$ $$T_{2s} = T_{min}\, r^x$$

$$(\Delta T_s)_{comp} = T_{2s} - T_1 = T_{min}\, r^x - T_{min} = T_{min}\,(r^x - 1)$$

$\therefore$ $$(\Delta T)_{comp} = \frac{T_{min}\left(r^x - 1\right)}{\eta_C}$$

$$\frac{T_3}{T_{4s}} = \left(\frac{p_3}{p_4}\right)^{(\gamma-1)/\gamma} = \left(\frac{rp_1}{\sqrt{r}\cdot p_1}\right)^x\; r^{x/2}$$

$\therefore$ $$T_{4s} = T_3\, r^{-x/2} = T_{max}\, r^{-x/2}$$

$$(\Delta T_s)_{turb} = T_3 - T_{4s} = T_{max} - T_{max}\cdot r^{-x/2} = T_{max}\,(1 - r^{-x/2})$$

$\therefore$ $$(\Delta T)_{turb\,1} = \eta_T\, T_{max}\,(1 - r^{-x/2}) = (\Delta T)_{turb\,2}$$

$\therefore$ $$W_{net} = c_p\left[2\eta_T T_{max}\left(1 - r^{-x/2}\right) - \frac{T_{min}}{\eta_C}\left(r^x - 1\right)\right]$$

$$\frac{dW_{net}}{dr} = c_p\left[2\eta_T T_{max}\,\frac{x}{2}\cdot r^{-x/2-1} - \frac{T_{min}}{\eta_C}\,x\cdot r^{x-1}\right] = 0$$

On simplification

$$r^{3x/2} = \eta_T\,\eta_C\,\frac{T_{max}}{T_{min}}$$

$\therefore$ $$r_{opt} = \left(\eta_T\eta_C\,\frac{T_{max}}{T_{min}}\right)^{2\gamma/3(\gamma-1)}$$

(b) With reference to Fig. Ex. 13.9(b)

$$\frac{T_{2s}}{T_{min}} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = \left(\sqrt{r}\right)^x = r^{x/2}$$

$$(\Delta T_s)_{comp\,1} = T_{2s} - T_1 = T_{min}\,(r^{x/2} - 1)$$

$$(\Delta T)_{comp1} = \frac{T_{min}(r^{x/2} - 1)}{\eta_C} = (\Delta T)_{comp\,2}$$

$$\frac{T_{max}}{T_{6s}} = \left(\frac{p_5}{p_6}\right)^{(\gamma-1)/\gamma} = r^x$$

$\therefore$ $$T_{6s} = T_{max}\cdot r^{-x}$$

$$(\Delta T_s)_{turb} = T_{max} - T_{6s} = T_{max}\,(1 - r^{-x})$$

$$(\Delta T)_{turb} = \eta_T\, T_{max}\,(1 - r^{-x})$$

$$W_{net} = c_p \left[\eta_T T_{max} \left(1 - r^{-x}\right) - \frac{2T_{min}\left(r^{x/2} - 1\right)}{\eta_C} \right]$$

$$\frac{dW_{net}}{dr} = c_p \left[\eta_T T_{max}\, x \cdot r^{-(x+1)} - \frac{2T_{min}}{\eta_C} \frac{x}{2} \cdot r^{(x/2)-1} \right] = 0$$

on simplification $$r_{opt} = \left(\eta_T \eta_C \frac{T_{max}}{T_{min}} \right)^{2\gamma/3(\gamma-1)}$$

This is the same as in (a).

If $$\eta_C = 0.85,\ \eta_T = 0.9$$

$$\frac{1}{x} = \frac{\gamma}{\gamma - 1} = \frac{1.4}{0.4} = \frac{7}{2}$$

$$\frac{T_{max}}{T_{min}} = 3.5$$

$\therefore$ $$r_{opt} = (0.85 \times 0.9 \times 3.5)^{2/3 \times 7/2} = 9.933$$ *Ans.*

$$W_{net}\,(a) = c_p \left[2\eta_T T_{max} \left(1 - r^{-x/2}\right) - \frac{T_{min}}{\eta_C} \left(r^x - 1\right) \right]$$

$$= c_p \left[2 \times 0.9 \times T_{max} \left(1 - 9.933^{-0.143}\right) - \frac{T_{min}}{0.85} \left(9.933^{0.286} - 1\right) \right]$$

$$= c_p \cdot T_{min} \left[2 \times 0.9 \times 3.5 \left(1 - \frac{1}{1.388}\right) - T\,1.178\,(0.928) \right]$$

$$= 0.670\, c_p \cdot T_{min}$$

$$W_{net}\,(b) = c_p\, T_{min} \left[0.9 \times 3.5 \left(1 - 9.933^{-0.286}\right) - \frac{2}{0.85} \left(9.933^{0.143} - 1\right) \right]$$

$$= c_p\, T_{min}\,(1.518 - 0.914) = 0.604\, c_p\, T_{min}$$

$$\frac{W_{net}(a) - W_{net}(b)}{W_{net}(a)} \times 100$$

$$= \frac{0.670 - 0.604}{0.670} \times 100 = 10.9\%$$ **Proved.**

***Example* 13.10**

A turbojet aircraft flies with a velocity of 300 *m/s at an altitude where the air is at* 0.35 *bar and* – 40°C. *The compressor has a pressure ratio of* 10, *and the temperature of the gases at the turbine inlet is* 1100°C. *Air enters the compressor at a rate of* 50 *kg/s. Estimate (a) the temperature and pressure of the gases at the turbine exit, (b) the velocity of gases at the nozzle exit, and (c) the propulsive efficiency of the cycle.*

Solution (a) For isentropic flow of air in the diffusor (Fig. Ex. 13.10)

$$\overset{=0}{\cancel{Q}_{1-2}} - \overset{=0}{\cancel{W}_{1-2}} = h_2 - h_1 + \frac{\overset{=0}{\cancel{V_2^2}} - V_1^2}{2}$$

$$0 = c_p (T_2 - T_1) - \frac{V_1^2}{2}$$

$$T_2 = T_1 + \frac{V_1^2}{2c_p} = 233 + \frac{300^2}{2 \times 1.005} \times 10^{-3}$$

$$= 277.78 \text{ K}$$

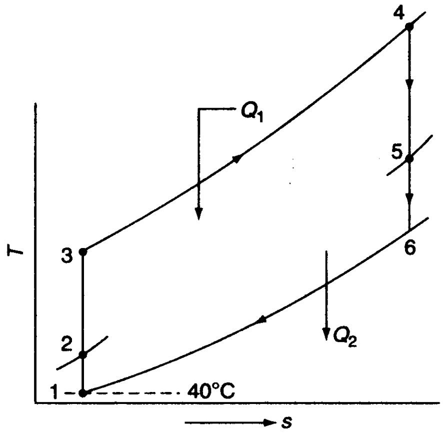
$$p_2 = p_1 (T_2/T_1)^{\gamma/(\gamma-1)}$$

$$= 35 \frac{\text{kN}}{\text{m}^2}\left(\frac{277.78}{233}\right)^{1.4/0.4} = 64.76 \text{ kPa}$$

$$p_3 = r_p p_2 = 10 \times 64.76 = 647.6 \text{ kPa}$$

$$T_3 = \left(\frac{p_3}{p_2}\right)^{(\gamma-1)/\gamma} T_2 = 277.78\,(10)^{0.4/1.4}$$

$$= 536.66 \text{ K}$$



[redacted]

$$W_C = W_T$$

$$h_3 - h_2 = h_4 - h_5$$

or, $$T_3 - T_2 = T_4 - T_5$$

$$T_5 = T_4 - T_3 + T_2 = 1373 - 536.66 + 277.78 = 1114.12 \text{ K}$$

$$p_5 = \left(\frac{T_5}{T_4}\right)^{\gamma/(\gamma-1)} p_4 = 647.6\left(\frac{1114.12}{1373}\right)^{3.5} = 311.69 \text{ K}$$ *Ans.* (a)

(b) For isentropic expansion of gases in the nozzle,

$$T_6 = T_5\left(\frac{p_6}{p_5}\right)^{(\gamma-1)/\gamma} = 1114.12\left(\frac{35}{311.69}\right)^{0.286} = 596.12 \text{ K}$$

Neglecting the K.E. of gas at nozzle inlet,

$$V_6 = [2c_p (T_5 - T_6) \times 1000]^{1/2} = [2 \times 1.005\,(1114.12 - 596.12) \times 1000]^{1/2}$$

$$= 1020.4 \text{ m/s}$$ *Ans.* (b)

(c) The propulsive efficiency of a turbojet engine is the ratio of the propulsive power developed $\dot{W}_P$ to the total heat transfer to the fluid.

$$\dot{W}_P = w[V_{exit} - V_{inlet}]\, V_{aircraft} = 50\,[1020.4 - 300] \times 300 \,\frac{\text{kg}}{\text{s}} \times \frac{\text{m}^2}{\text{s}^2} = 10.806 \text{ MW}$$

$$Q_1 = w\,(h_4 - h_3) = 50 \times 1.005\,(1373 - 536.66) = 42.026 \text{ MW}$$

$$\eta_P = \frac{10.806}{42.026} = 0.257 \text{ or } 25.7\%$$ *Ans.*

Example 13.11

In a combined GT – ST plant, the exhaust gas from the GT is the supply gas to the steam generator at which a further supply of fuel is burned in the gas. The pressure ratio for the GT is 8, the inlet air temperature is 15°C and the maximum cycle temperature is 800°C.

Combustion in the steam generator raises the gas temperature to 800°C and the gas leaves the generator at 100°C. The steam condition at supply is at 60 bar, 600°C and the condenser pressure is 0.05 bar. Calculate the flow rates of air and steam required for a total power output of 190 MW and the overall efficiency of the combined plant. What would be the air-fuel ratio? Assume ideal processes. Take $c_{p_g} = 1.11$ and $c_{p_a} = 1.005$ kJ/kg K, $\gamma_g = 1.33$, $\gamma_a = 1.4$, C.V. of fuel = 43.3 MJ/kg. Neglect the effect of fuel flow on the total mass flow of gas expanding in the gas turbine.

Solution With reference to Fig. 13.32.

$$T_b = T_a (p_b/p_a]^{(\gamma-1)/\gamma} = 288 \times 8^{1.4/0.4} = 522 \text{ K}$$

$$T_d = \frac{T_C}{r_p^{(\gamma-1)/\gamma}} = \frac{1073}{8^{0.33/1.33}} = \frac{1073}{1.682} = 638 \text{ K}$$

$$W_{GT} = c_{p_g}(T_c - T_d) - c_{p_a}(T_b - T_a)$$

$$= 1.11\,(1073 - 638) - 1.005\,(522 - 288) = 249 \text{ kJ/kg}$$

$$Q_1 = c_{p_g}(T_c - T_b) = 1.11\,(1073 - 522) = 612 \text{ kJ/kg}$$

$$Q'_1 = c_{p_g}(T_e - T_d) = 1.11(1073 - 638)$$

$$= 483 \text{ kJ/kg}$$

$$h_1 = 3775,\ h_2 = 2183,\ h_3 = 138 = h_4, \text{ all in kJ/kg}$$

$$(Q_1)_{St} = h_1 - h_3 = 3775 - 138 = 3637 \text{ kJ/kg}$$

$$Q''_{fe} = 1.11(800 - 100) = 777 \text{ kJ/kg}$$

By energy balance of the steam generator,

$$w_a \times 777 = w_s \times 3637$$

$$w_a/w_s = 4.68$$

$$W_{ST} = h_1 - h_2 = 3775 - 2183 = 1592 \text{ kJ/kg}$$

$$w_a \times 249 + w_s \times 1592 = 190 \times 10^3 \text{ kW}$$

$$w_s(4.68 \times 249 + 1592) = w_s \times 2.757 \times 10^3 = 190 \times 10^3$$

$$w_s = 68.9 \text{ kg/s and } w_a = 322.5 \text{ kg/s}$$

Now,

$$w_a(612 + 483) = w_f \times 43{,}300$$

$$w_a/w_f = \text{A/F ratio} = \frac{43{,}300}{1095} = 39.5 \qquad \textit{Ans.}$$

$$\text{Fuel energy input} = \frac{322.5}{39.5} \times 43{,}300 = 353525 \text{ kW} = 353.5 \text{ MW}$$

$$\eta_{0A} = \frac{190}{353.5} = 0.537 \text{ or } 53.7\% \qquad \textit{Ans.}$$

Review Questions

13.1 What are cyclic and non-cyclic heat engines? Give examples.

13.2 What are the four processes which constitute the Stirling cycle? Show that the regenerative Stirling cycle has the same efficiency as the Carnot cycle.

13.3 State the four processes that constitute the Ericsson cycle. Show that the regenerative Ericsson cycle has the same efficiency as the Carnot cycle.

13.4 Mention the merits and demerits of the Stirling and Ericsson cycles.

13.5 What is an air standard cycle? Why are such cycles conceived?

13.6 What is a spark ignition engine? What is the air standard cycle of such an engine? What are its four processes?

13.7 Show that the efficiency of the Otto cycle depends only on the compression ratio.

13.8 How is the compression ratio of an SI engine fixed?

13.9 What is a compression ignition engine? Why is the compression ratio of such an engine more than that of an SI engine?

13.10 State the four processes of the Diesel cycle.

13.11 Explain the mixed or dual cycle.

13.12 For the same compression ratio and heat rejection, which cycle is most efficient: Otto, Diesel or Dual? Explain with p–v and T–s diagrams.

13.13 With the help of p–v and T–s diagrams, show that for the same maximum pressure and temperature of the cycle and the same heat rejection,

$$\eta_{\text{Diesel}} > \eta_{\text{Dual}} > \eta_{\text{Otto}}$$

13.14 What are the three basic components of a gas turbine plant? What is the air standard cycle of such a plant? What are the processes it consists of?

13.15 Show that the efficiency of the Brayton cycle depends only on the pressure ratio.

13.16 What is the application of the closed cycle gas turbine plant?

13.17 Discuss the merits and demerits of Brayton and Otto cycles applied to reciprocating and rotating plants.

13.18 What is the effect of regeneration on Brayton cycle efficiency? Define the effectiveness of a regenerator.

13.19 What is the effect of irreversibilities in turbine and compressor on Brayton cycle efficiency?

13.20 Explain the effect of pressure ratio on the net output and efficiency of a Brayton cycle.

13.21 Drive the expression of optimum pressure ratio for maximum net work output in an ideal Brayton cycle. What is the corresponding cycle efficiency.

13.22 Explain the effects of: (a) intercooling, and (b) reheating, on Brayton cycle.

13.23 What is a free shaft turbine?

13.24 With the help of flow and T–s diagrams explain the air standard cycle for a jet propulsion plant.

13.25 With the help of a neat sketch explain the operation of a turbojet engine. How is the thrust developed in this engine? Why does a commercial airplane fly at high altitudes?

13.26 Define propulsive power and propulsive efficiency.

13.27 Why are regenerators and intercoolers not used in aircraft engines? What is after-burning? Why is it used?

13.28 Explain the working of a turbofan engine with the help of a neat sketch. Define "bypass ratio". How does it influence the engine thrust?

13.29 How does a turboprop engine differ from a turbofan engine?

13.30 What is a ramjet? How is the thrust produced here?

13.31 What is a rocket? How is it propelled?

13.32 Explain the advantages and disadvantages of a gas turbine plant for a utility system.

13.33 What are the advantages of a combined gas turbine-steam turbine power plant?

13.34 With the help of flow and T–s diagrams explain the operation of a combined GT–ST plant. Why is supplementary firing often used?

Problems

13.1 In a Stirling cycle the volume varies between 0.03 and 0.06 m^3, the maximum pressure is 0.2 MPa, and the temperature varies between 540°C and 270°C. The working fluid is air (an ideal gas). (a) Find the efficiency and the work done per cycle for the simple cycle. (b) Find the efficiency and the work done per cycle for the cycle with an ideal regenerator, and compare with the Carnot cycle having the same isothermal heat supply process and the same temperature range. *Ans.* (a) 27.7%, 53.7 kJ/kg, (b) 32.2%

13.2 An Ericsson cycle operating with an ideal regenerator works between 1100 K and 288 K. The pressure at the beginning of isothermal compression is 1.013 bar. Determine (a) the compressor and turbine work per kg of air, and (b) the cycle efficiency. *Ans.* (a) $W_T = 465$ kJ/kg, $W_C = 121.8$ kJ/kg (b) 0.738

13.3 Plot the efficiency of the air standard Otto cycle as a function of the compression ratio for compression ratios from 4 to 16.

13.4 Find the air standard efficiencies for Otto cycles with a compression ratio of 6 using ideal gases having specific heat ratios 1.3, 1.4 and 1.67. What are the advantages and disadvantages of using helium as the working fluid?

13.5 An engine equipped with a cylinder having a bore of 15 cm and a stroke of 45 cm operates on an Otto cycle. If the clearance volume is 2000 cm^3, compute the air standard efficiency. *Ans.* 47.4%

13.6 In an air standard Otto cycle the compression ratio is 7, and compression begins at 35°C, 0.1 MPa. The maximum temperature of the cycle is 1100°C. Find (a) the temperature and pressure at the cardinal points of the cycle, (b) the heat supplied per kg of air, (c) the work done per kg of air, (d) the cycle efficiency, and (e) the m.e.p. of the cycle.

13.7 An engine working on the Otto cycle has an air standard cycle efficiency of 56% and rejects 544 kJ/kg of air. The pressure and temperature of air at the beginning of compression are 0.1 MPa and 60°C respectively. Compute (a) the compression ratio of the engine, (b) the work done per kg of air, (c) the pressure and temperature at the end of compression, and (d) the maximum pressure in the cycle.

13.8 For an air standard Diesel cycle with a compression ratio of 15, plot the efficiency as a function of the cut-off ratio for cut-off ratios from 1 to 4. Compare with the results of Problem 13.3.

13.9 In an air standard Diesel cycle, the compression ratio is 15. Compression begins at 0.1 MPa, 40°C. The heat added is 1.675 MJ/kg. Find (a) the maximum temperature of the cycle, (b) the work done per kg of air, (c) the cycle efficiency, (d) the temperature at the end of the isentropic expansion, (e) the cut-off ratio, (f) the maximum pressure of the cycle, and (g) the m.e.p. of the cycle.

13.10 Two engines are to operate on Otto and Diesel cycles with the following data:
Maximum temperature 1400 K, exhaust temperature 700 K. State of air at the beginning of compression 0.1 MPa, 300 K.
Estimate the compression ratios, the maximum pressures, efficiencies, and rate of work outputs (for 1 kg/min of air) of the respective cycles.
Ans. Otto—$r_k = 5.656$, $p_{max} = 2.64$ MPa, $W =$ 2872 kJ/kg, $\eta = 50\%$
Diesel—$r_k = 7.456$, $p_{max} = 1.665$ MPa, $W = 446.45$ kJ/kg, $\eta = 60.8\%$

13.11 An air standard limited pressure cycle has a compression ratio of 15 and compression begins at 0.1 MPa, 40°C. The maximum pressure is limited to 6 MPa and the heat added is 1.675 MJ/kg. Compute (a) the heat supplied at constant volume per kg of air, (b) the heat supplied at constant pressure per kg of air, (c) the work done per kg of air, (d) the cycle efficiency, (e) the temperature at the end of the constant volume heating process, (f) the cut-off ratio, and (g) the m.e.p. of the cycle. *Ans.* (a) 235 kJ/kg, (b) 1440 kJ/kg, (c) 1014 kJ/kg, (d) 60.5%, (e) 1252 K, (f) 2.144, (g) 1.21 MPa

13.12 In an ideal cycle for an internal combustion engine the pressure and temperature at the beginning of adiabatic compression are respectively 0.11 MPa and 115°C, the compression ratio being 16. At the end of compression heat is added to the working fluid, first, at constant volume, and then at constant pressure reversibly. The working fluid is then expanded adiabatically and reversibly to the original volume.

If the working fluid is air and the maximum pressure and temperature are respectively 6 MPa and 2000°C, determine, per kg of air (a) the pressure, temperature, volume, and entropy of the air at the five cardinal points of the cycle (take s_1 as the entropy of air at the beginning of compression), and (b) the work output and efficiency of the cycle.

13.13 Show that the air standard efficiency for a cycle comprising two constant pressure processes and two isothermal processes (all reversible) is given by

$$\eta = \frac{(T_1 - T_2)\ \ln\left(r_p\right)^{(\gamma-1)\gamma}}{T_1\left[1+\ln\left(r_p\right)^{(\gamma-1)/\gamma} - T_2\right]}$$

where T_1 and T_2 are the maximum and minimum temperatures of the cycle, and r_p is the pressure ratio.

13.14 Obtain an expression for the specific work done by an engine working on the Otto cycle in terms of the maximum and minimum temperatures of the cycle, the compression ratio r_k, and constants of the working fluid (assumed to be an ideal gas).

Hence show that the compression ratio for maximum specific work output is given by

$$r_k = \left(\frac{T_{min}}{T_{max}}\right)^{1/2(1-\gamma)}$$

13.15 A dual combustion cycle operates with a volumetric compression ratio $r_k = 12$, and with a cut-off ratio 1.615. The maximum pressure is given by $p_{max} = 54p_1$, where p_1 is the pressure before compression. Assuming indices of compression and expansion of 1.35, show that the m.e.p. of the cycle

$$p_m = 10\,p_1$$

Hence evaluate (a) temperatures at cardinal points with $T_1 = 335$ K, and (b) cycle efficiency.

Ans. (a) $T_2 = 805$ K, $p_2 = 29.2\,p_1$, $T_3 = 1490$ K, $T_4 = 2410$ K, $T_5 = 1200$ K, (b) $\eta = 0.67$

13.16 Recalculate (a) the temperatures at the cardinal points, (b) the m.e.p., and (c) the cycle efficiency when the cycle of Problem 13.15 is a Diesel cycle with the same compression ratio and a cut-off ratio such as to give an expansion curve coincident with the lower part of that of the dual cycle of Problem 13.15. *Ans.* (a) $T_2 = 805$ K, $T_3 = 1970$ K, $T_4 = 1142$ K (b) $6.82\,p_1$, (c) $\eta = 0.513$

13.17 In an air standard Brayton cycle the compression ratio is 7 and the maximum temperature of the cycle is 800°C. The compression begins at 0.1 MPa, 35°C. Compare the maximum specific volume and the maximum pressure with the Otto cycle of Problem 13.6. Find (a) the heat supplied per kg of air, (b) the net work done per kg of air, (c) the cycle efficiency, and (d) the temperature at the end of the expansion process.

13.18 A gas turbine plant operates on the Brayton cycle between the temperatures 27°C and 800°C. (a) Find the pressure ratio at which the cycle efficiency approaches the Carnot cycle efficiency, (b) find the pressure ratio at which the work done per kg of air is maximum, and (c) compare the efficiency at this pressure ratio with the Carnot efficiency for the given temperatures.

13.19 In a gas turbine plant working on the Brayton cycle the air at the inlet is at 27°C, 0.1 MPa. The pressure ratio is 6.25 and the maximum temperature is 800°C. The turbine and compressor efficiencies are each 80%. Find (a) the compressor work per kg of air, (b) the turbine work per kg of air, (c) the heat supplied per kg of air, (d) the cycle efficiency, and (e) the turbine exhaust temperature.

Ans. (a) 259 kJ/kg, (b) 351.68 kJ/kg, (c) 569.43 kJ/kg, (d) 16.2%, (e) 723 K

13.20 Solve Problem 13.19 if a regenerator of 75% effectiveness is added to the plant.

13.21 Solve Problem 13.19 if the compression is divided into two stages, each of pressure ratio 2.5 and efficiency 80%, with intercooling to 27°C.

13.22 Solve Problem 13.21 if a regenerator of 75% effectiveness is added to the plant.

13.23 Solve Problem 13.19 if a reheat cycle is used. The turbine expansion is divided into two stages, each of pressure ratio 2.5 and efficiency 80%, with reheat to 800°C.

13.24 Solve Problem 13.23 if a regenerator of 75% effectiveness is added to the plant.

13.25 Solve Problem 13.24 if the staged compression of Problem 13.21 is used in the plant.

13.26 Find the inlet condition for the free-shaft turbine if a two-shaft arrangement is used in Problem 13.19.

13.27 A simple gas turbine plant operating on the Brayton cycle has air inlet temperature 27°C, pressure ratio 9, and maximum cycle temperature 727°C.

What will be the improvement in cycle efficiency and output if the turbine process is divided into two stages each of pressure ratio 3, with intermediate reheating to 727°C? *Ans.* – 18.3%, 30.6%

13.28 Obtain an expression for the specific work output of a gas turbine unit in terms of pressure ratio, isentropic efficiencies of the compressor and turbine, and the maximum and minimum temperatures, T_3 and T_1. Hence show that the pressure ratio r_p for maximum power is given by

$$r_p = \left(\eta_T \eta_C \frac{T_3}{T_1} \right)^{\gamma/2(\gamma-1)}$$

If T_3 = 1073 K, T_1 = 300 K, η_C = 0.8, η_T = 0.8 and γ = 1.4 compute the optimum value of pressure ratio, the maximum net work output per kg of air, and corresponding cycle efficiency. *Ans.* 4.263, 100 kJ/kg, 17.2%

13.29 A gas turbine plant draws in air at 1.013 bar, 10°C and has a pressure ratio of 5.5. The maximum temperature in the cycle is limited to 750°C.

Compression is conducted in an uncooled rotary compressor having an isentropic efficiency of 82%, and expansion takes place in a turbine with an isentropic efficiency of 85%. A heat exchanger with an efficiency of 70% is fitted between the compressor outlet and combustion chamber. For an air flow of 40 kg/s, find (a) the overall cycle efficiency, (b) the turbine output, and (c) the air-fuel ratio if the calorific value of the fuel used in 45.22 MJ/kg. *Ans.* (a) 30.4%, (b) 4272 kW, (c) 115

13.30 A gas turbine for use as an automotive engine is shown in Fig. P 13.30. In the first turbine, the gas expands to just a low enough pressure p_5, for the turbine to drive the compressor. The gas is then expanded through a second turbine connected to the drive wheels. Consider air as the working fluid, and assume that all processes are ideal. Determine (a) pressure p_5, (b) the net work per kg and mass flow rate, (c) temperature T_3 and cycle thermal efficiency, and (d) the *T-s* diagram for the cycle.

13.31 Repeat Problem 13.30 assuming that the compressor has an efficiency of 80%, both the turbines have efficiencies of 85%, and the regenerator has an efficiency of 72%.

13.32 An ideal air cycle consists of isentropic compression, constant volume heat transfer, isothermal expansion to the original pressure, and constant pressure heat transfer to the original temperature. Deduce an expression for the cycle efficiency in terms of volumetric compression ratio r_k, and isothermal expansion ratio, r_e. In such a cycle, the pressure and temperature at the start of compression are 1 bar and 40°C, the compression ratio is 8, and the maximum pressure is 100 bar. Determine the cycle efficiency and the m.e.p.

Ans. 51.5%, 3.45 bar

13.33 For a gas turbine jet propulsion unit, shown in Fig. 13.24, the pressure and temperature entering the compressor are 1 atm and 15°C respectively. The pressure ratio across the compressor is 6 to 1 and the temperature at the turbine inlet is 1000°C. On leaving the turbine the air enters the nozzle and expands to 1 atm. Determine the pressure at the nozzle inlet and the velocity of the air leaving the nozzle.

13.34 Repeat Problem 13.33, assuming that the efficiency of the compressor and turbine are both 85%, and that the nozzle efficiency is 95%.

13.35 Develop expressions for work output per kg and the efficiency of an ideal Brayton cycle with regeneration, assuming maximum possible regeneration. For fixed maximum and minimum temperatures, how do the efficiency and work outputs vary with the pressure ratio? What is the optimum pressure ratio?

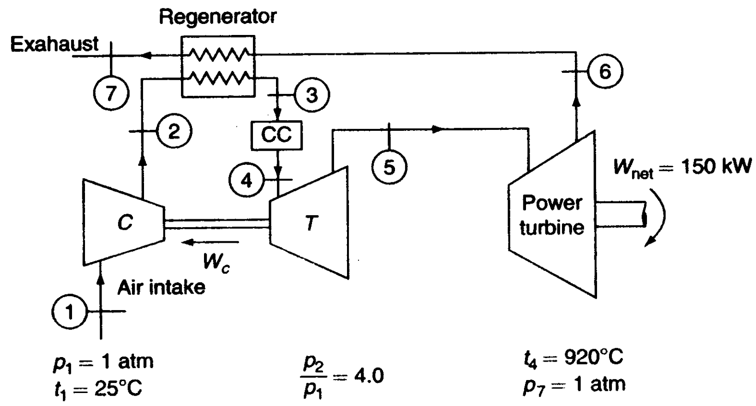
13.36 For an air standard Otto cycle with fixed intake and maximum temperatures, T_1 and T_3, find the compression ratio that renders the net work per cycle a maximum. Derive the expression for cycle efficiency at this compression ratio.

If the air intake temperature, T_1, is 300 K and the maximum cycle temperature, T_3, is 1200 K, compute the compression ratio for maximum net work, maximum work output per kg in a cycle, and the corresponding cycle efficiency.

Find the changes in work output and cycle efficiency when the compression ratio is increased from this optimum value to 8. Take c_v = 0.718 kJ/kg K.

Ans. 5.65, 215 kJ/kg, 50%, ΔW_{net} = – 9 kJ/kg, $\Delta\eta$ = + 6.4%

13.37 Show that the mean effective pressure, p_m, for the Otto cycle is given by



[redacted]

$$p_m = \frac{\left(p_3 - p_1 r_k^{\gamma}\right)\left(1 - \frac{1}{r_k^{\gamma-1}}\right)}{(\gamma - 1)(r_k - 1)}$$

where $p_3 = p_{max}$, $p_1 = p_{min}$ and r_k is the compression ratio.

13.38 A gas turbine plant operates on the Brayton cycle using an optimum pressure ratio for maximum net work output and a regenerator of 100% effectiveness.

Derive expressions for net work output per kg of air and corresponding efficiency of the cycle in terms of the maximum and the minimum temperatures.

If the maximum and minimum temperatures are 800°C and 30°C respectively, compute the optimum value of pressure ratio, the maximum net work output per kg and the corresponding cycle efficiency.

$$Ans. \; (W_{net})_{max} = c_p \left(\sqrt{T_{max}} - \sqrt{T_{min}}\right)^2$$

$$(\eta_{cycle})_{max} = 1 - \sqrt{\frac{T_{min}}{T_{max}}}, \; (r_p)_{opt} = 9.14,$$

$$(W_{net})_{max} = 236.79 \text{ kJ/kg}; \; \eta_{cycle} = 0.469$$

13.39 A gas turbine unit is to provide peaking power for an electrical utility with a net power output of 10 MW. The pressure ratio across the compressor is 7, the efficiency of the compressor 80%, and the efficiency of the turbine is 92%. In order to conserve fuel, a regenerator with an effectiveness of 85% is used. The maximum temperature of the cycle is 1200 K. The air at compressor inlet is at 20°C, 1.1 bar. Assume the working fluid to be air which behaves as an ideal gas with $c_p = 1.005$ kJ/kg K and $\gamma = 1.4$. Neglect pressure drops in the combustion chamber and regenerator. Determine the required air flow and the fuel flow rates for a fuel heating value of 42 MJ/kg, and the power plant efficiency.

13.40 Show that for the Stirling cycle with all the processes occurring reversibly but where the heat rejected is not used for regenerative heating, the efficiency is given by

$$\eta = 1 - \frac{\left(\frac{T_1}{T_2} - 1\right) + (\gamma - 1)\ln r}{\left(\frac{T_1}{T_2} - 1\right) + (\gamma - 1)\frac{T_1}{T_2}\ln r}$$

where r is the compression ratio and T_1/T_2 the maximum to minimum temperature ratio.

Determine the efficiency of this cycle using hydrogen ($R = 4.307$ kJ/kg K, $c_p = 14.50$ kJ/kg K) with a pressure and temperature prior to isothermal compression of 1 bar and 300 K respectively, a maximum pressure of 2.55 MPa and heat supplied during the constant volume heating of 9300 kJ/kg. If the heat rejected during the constant volume cooling can be utilized to provide the constant volume heating, what will be the cycle efficiency? Without altering the temperature ratio, can the efficiency be further improved in the cycle?

13.41 Helium is used as the working fluid in an ideal Brayton cycle. Gas enters the compressor at 27°C and 20 bar and is discharged at 60 bar. The gas is heated to 1000°C before entering the turbine. The cooler returns the hot turbine exhaust to the temperature of the compressor inlet. Determine: (a) the temperatures at the end of compression and expansion, (b) the heat supplied, the heat rejected and the net work per kg of He, and (c) the cycle efficiency and the heat rate. Take $c_p = 5.1926$ kJ/kg K.

Ans. (a) 465.5, 820.2 K, (b) 4192.5, 2701.2, 1491.3 kJ/kg, (c) 0.3557, 10,121 kJ/kWh

13.42 In an air standard cycle for a gas turbine jet propulsion unit, the pressure and temperature entering the compressor are 100 kPa and 290 K, respectively. The pressure ratio across the compressor is 6 to 1 and the temperature at the turbine inlet is 1400 K. On leaving the turbine the air enters the nozzle and expands to 100 kPa. Assuming that the efficiency of the compressor and turbine are both 85% and that the nozzle efficiency is 95%, determine the pressure at the nozzle inlet and the velocity of the air leaving the nozzle. *Ans.* 285 kPa, 760 m/s

13.43 A stationary gas turbine power plant operates on the Brayton cycle and delivers 20 MW to an electric generator. The maximum temperature is 1200 K and the minimum temperatures is 290 K. The minimum pressure is 95 kPa and the maximum pressure is 380 kPa. If the isentropic efficiencies of the turbine and compressor are 0.85 and 0.80 respectively, find (a) the mass flow rate of air to the compressor, (b) the volume flow rate of air to the compressor, (c) the fraction of the turbine work output needed to drive the compressor, (d) the cycle efficiency.

If a regenerator of 75% effectiveness is added to the plant, what would be the changes in the cycle efficiency and the net work output? *Ans.* (a) 126.37 kg/s, (b) 110.71 m³/s, (c) 0.528, (d) 0.2146, $\Delta\eta = 0.148$ $\Delta W_{net} = 0$

13.44 Air enters the compressor of a gas turbine operating on Brayton cycle at 1 bar, 27°C. The pressure ratio in the cycle is 6. Calculate the maximum temperature in the cycle and the cycle efficiency. Assume $W_T = 2.5\ W_C$ and $\gamma = 1.4$. *Ans.* 1251.4 K, 40%

13.45 In an ideal jet propulsion cycle, air enters the compressor at 1 atm, 15°C. The pressure of air leaving the compressor is 5 atm and the maximum temperature is 870°C. The air expands in the turbine to such a pressure that the turbine work is equal to the compressor work. On leaving the turbine the air expands reversibly and adiabatically in a nozzle to 1 atm. Determine the velocity of air leaving the nozzle. *Ans.* 710.3 m/s

13.46 In a gas turbine the compressor is driven by the h.p. turbine. The exhaust from the h.p. turbine goes to a free-shaft l.p. turbine which runs the load. The air flow rate is 20 kg/s, and the minimum and maximum temperatures are respectively 300 K and 1000 K. The compressor pressure ratio is 4. Calculate the pressure ratio of the l.p. turbine and the temperature of the exhaust gases from the unit. The compressor and turbine are isentropic. Take c_p of air and exhaust gases as 1 kJ/kg K and $\gamma = 1.4$. *Ans.* 2.3, 673 K

13.47 A regenerative gas turbine with intercooling and reheat operates at steady state. Air enters the compressor at 100 kPa, 300 K with a mass flow rate of 5.807 kg/s. The pressure ratio across the two-stage compressor as well as the turbine is 10. The intercooler and reheater each operate at 300 kPa. At the inlets to the turbine stages, the temperature is 1400 K. The temperature at inlet to the second compressor stage is 300 K. The efficiency of each compressor and turbine stage is 80%. The regenerator effectiveness is 80%. Determine (a) the thermal efficiency, (b) the back work ratio, W_C/W_T, (c) the net power developed. *Ans.* (a) 0.443, (b) 0.454, (c) 2046 kW

13.48 In a regenerative gas turbine power plant air enters the compressor at 1 bar, 27°C and is compressed to 4 bar. The isentropic efficiency of the compressor is 80% and the regenerator effectiveness is 90%. All of the power developed by the h.p. turbine is used to drive the compressor and the l.p. turbine provides the net power output of 97 kW. Each turbine has an isentropic efficiency of 87% and the temperature at inlet to the h.p. turbine is 1200 K. Determine (a) the mass flow rate of air into the compressor, (b) the thermal efficiency, (c) the temperature of the air at the exit of the regenerator. *Ans.* (a) 0.562 kg/s, (b) 0.432, (c) 523.2 K

CHAPTER 14

Refrigeration Cycles

14.1 REFRIGERATION BY NON-CYCLIC PROCESSES

Refrigeration is the cooling of a system below the temperature of its surroundings.

The melting of ice or snow was one of the earliest methods of refrigeration and is still employed. Ice melts at 0°C. So when ice is placed in a given space warmer than 0°C, heat flows into the ice and the space is cooled or refrigerated. The latent heat of fusion of ice is supplied from the surroundings, and the ice changes its state from solid to liquid.

Another medium of refrigeration is solid carbon dioxide or dry ice. At atmospheric pressure CO_2 cannot exist in a liquid state, and consequently, when solid CO_2 is exposed to atmosphere, it sublimates, i.e. it goes directly from solid to vapour, by absorbing the latent heat of sublimation (620 kJ/kg at 1 atm, −78.5°C from the surroundings (Fig. 14.1). Thus dry ice is suitable for low temperature refrigeration.

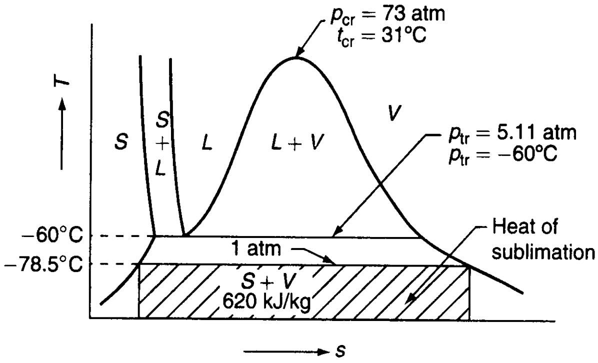


Fig. 14.1 *T-s diagram of CO_2*

In these two examples it is observed that the refrigeration effect has been accomplished by non-cyclic processes. Of greater importance, however, are the methods in which the cooling substance is not consumed and discarded, but used again and again in a thermodynamic cycle.

14.2 REVERSED HEAT ENGINE CYCLE

A reversed heat engine cycle, as explained in Sec. 6.12, is visualized as an engine operating in the reverse way, i.e. receiving heat from a low temperature region, discharging heat to a high temperature region, and receiving a net inflow of work (Fig. 14.2). Under such conditions the cycle is called a *heat pump cycle* or a *refrigeration cycle* (see Sec. 6.6). For a heat pump

$$(\text{COP})_{\text{H.P.}} = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2}$$

and for a refrigerator

$$(\text{COP})_{\text{ref}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

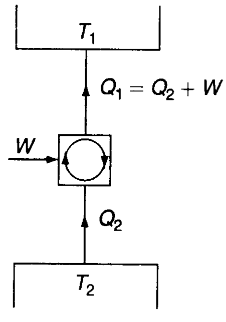


Fig. 14.2 *Reversed heat engine cycle*

The working fluid in a refrigeration cycle is called a *refrigerant*. In the reversed Carnot cycle (Fig. 14.3), the refrigerant is first compressed reversibly and adiabatically in process 1–2 where the work input per kg of refrigerant is W_c, then it is condensed reversibly in process 2–3 where the heat rejection is Q_1, the refrigerant then expands reversibly and adiabatically in process 3–4 where the work output is W_E, and finally it absorbs heat Q_2 reversibly by evaporation from the surroundings in process 4–1.

Here $$Q_1 = T_1(s_2 - s_3),\ Q_2 = T_2(s_1 - s_4)$$

and $$W_{net} = W_C - W_E = Q_1 - Q_2 = (T_1 - T_2)(s_1 - s_4)$$

where T_1 is the temperature of heat rejection and T_2 the temperature of heat absorption.

$$(\text{COP}_{ref})_{rev} = \frac{Q_2}{W_{net}} = \frac{T_2}{T_1 - T_2}$$

and $$(\text{COP}_{H.P.})_{rev} = \frac{Q_1}{W_{net}} = \frac{T_1}{T_1 - T_2} \tag{14.1}$$

As shown in Sec. 6.16, these are the maximum values for any refrigerator or heat pump operating between T_1 and T_2. It is important to note that for the same T_2 or T_1, the COP increases with the decrease in the temperature difference $(T_1 - T_2)$, i.e. the closer the temperatures T_1 and T_2, the higher the COP.

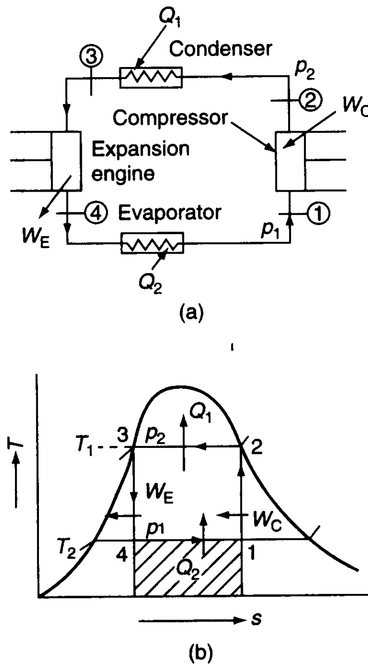


Fig. 14.3 Reversed carnot cycle

14.3 VAPOUR COMPRESSION REFRIGERATION CYCLE

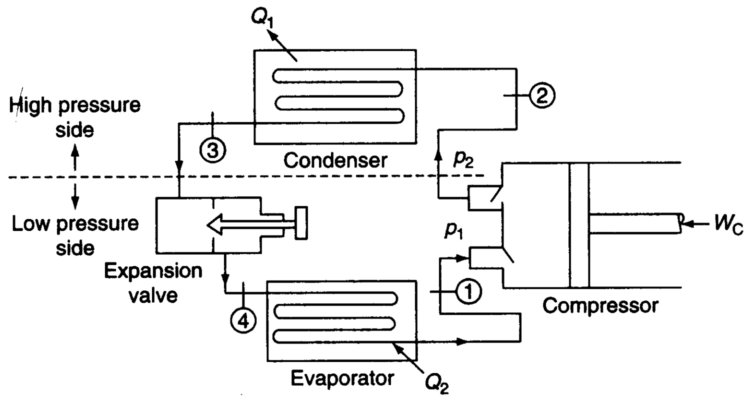
In an actual vapour refrigeration cycle, an expansion engine, as shown in Fig. 14.3, is not used, since power recovery is small and does not justify the cost of the engine. A throttling valve or a capillary tube is used for expansion in reducing the pressure from p_2 to p_1. The basic operations involved in a vapour compression refrigeration plant are illustrated in the flow diagram, Fig. 14.4, and the property diagrams, Fig. 14.5.

The operations represented are as follows for an idealized plant:

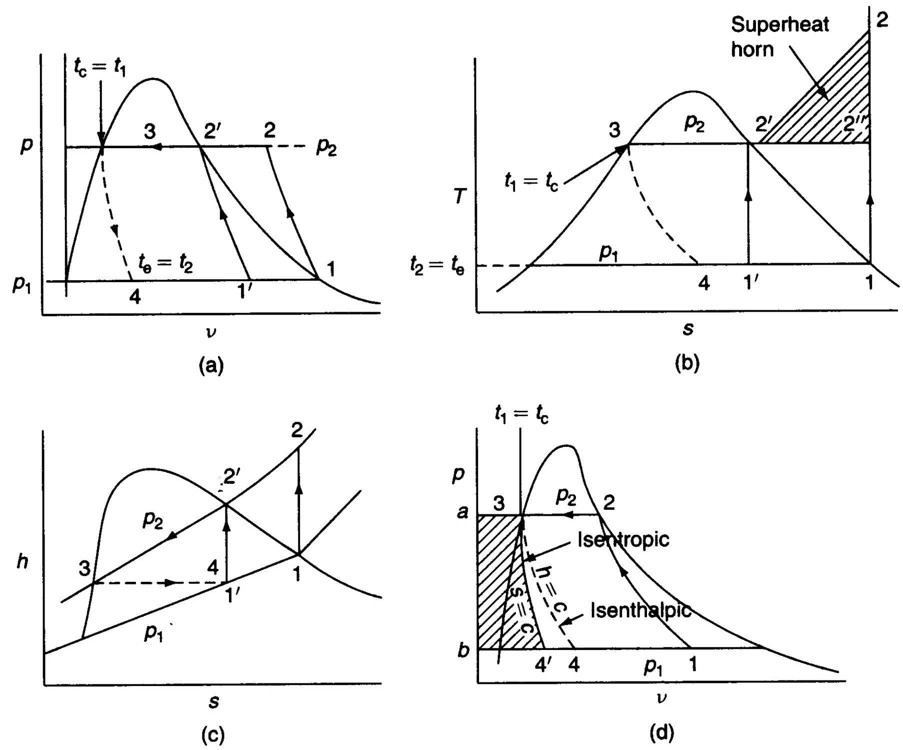
1. ***Compression*** A reversible adiabatic process 1–2 or 1′–2′ either starting with saturated vapour (state 1), called *dry compression*, or starting with wet vapour (state 1′), called *wet compression*. Dry compression (1–2) is always preferred to wet compression (1′–2′), because with wet compression there is a danger of the liquid refrigerant being trapped in the head of the cylinder by the rising piston which may damage the valves or the cylinder head, and the droplets of liquid refrigerant may wash away the lubricating oil from the walls of the cylinder, thus accelerating wear.

It is, therefore, desirable to have compression with vapour initially dry saturated at 1, as shown in Fig. 14.5(b) or even slightly superheated. The state of the vapour at the end of dry compression will be at 2 at pressure p_2 which is the saturation pressure of the refrigerant corresponding to the condensing temperature t_1, instead of being at t_2'', for the Carnot cycle. The increased work of the cycle due to the dry compression instead of wet compression is given by the area 2–2′–2″, known as *superheat horn* (Fig. 14.5(b))

2. ***Cooling and Condensing*** A reversible constant pressure process, 2–3, first desuperheated and then condensed, ending with saturated liquid. Heat Q_1 is transferred out.



Vapour compression refrigeration plant-flow diagram



Vapour compression refrigeration cycle: Throttling vs Isentropic expansion

3. ***Expansion*** An adiabatic throttling process 3–4, for which enthalpy remains unchanged. States 3 and 4 are equilibrium points. Process 3–4 is adiabatic (then only $h_3 = h_4$ by S.F.E.E.), but not isentropic.

$$T\mathrm{d}s = \mathrm{d}h - v\mathrm{d}p, \quad \text{or} \quad s_4 - s_3 = -\int_{p_1}^{p_2} \frac{v\mathrm{d}p}{T}$$

Hence it is irreversible and cannot be shown in property diagrams. States 3 and 4 have simply been joined by a dotted line.

The positive work recovered during the isentropic expansion in the expansion engine is given by the area $3-a-b-4'$ (Fig. 14.5(d)), which is much smaller than the compressor work given by the area $1-2-a-b-1$. In a steady flow process, the work done is given by $-\int v dp$. For the same pressure difference dp, the work depends on the volume v of the working fluid.

In the expander, the refrigerant is in the liquid phase, whereas in the compressor, it is in the gas phase. The volume of the vapour is very large compared to the volume of the liquid($v_g >> v_f$). Hence, the positive work of isentropic expansion is seldom large enough to justify the cost of the expander. Moreover, the friction and other losses may exceed the gain in work. Therefore, the isentropic expansion process of the Carnot cycle is replaced by a simple throttling or isenthalpic process by the use of a throttle valve or a capillary tube.

4. *Evaporation* A constant pressure reversible process, 4–1, which completes the cycle. The refrigerant is throttled by the expansion valve to a pressure, the saturation temperature at this pressure being below the temperature of the surroundings. Heat then flows, by virtue of temperature difference, from the surroundings, which gets cooled or refrigerated, to the refrigerant, which then evaporates, absorbing the latent heat of evaporation. The evaporator thus produces the cooling or the *refrigerating effect,* absorbing heat Q_2 from the surroundings by evaporation.

In refrigeration practice, enthalpy is the most sought-after property. The diagram in *p–h* coordinates is found to be the most con ,'nient. The constant property lines in the *p–h* diagram are shown in Fig. 14.6, and the vapour compression cycle in Fig. 14.7.

14.3.1 Performance and Capacity of a Vapour Compression Plant

Figure 14.8 shows the simplified diagram of a vapour compression refrigeration plant.

When steady state has been reached, for 1 kg flow of refrigerant through the cycle, the steady flow energy equations (neglecting K.E. and P.E. changes) may be written for each of the components in the cycle as given below.

Compressor $\qquad h_1 + W_c = h_2$

$\therefore \qquad W_c = (h_2 - h_1)$ kJ/kg

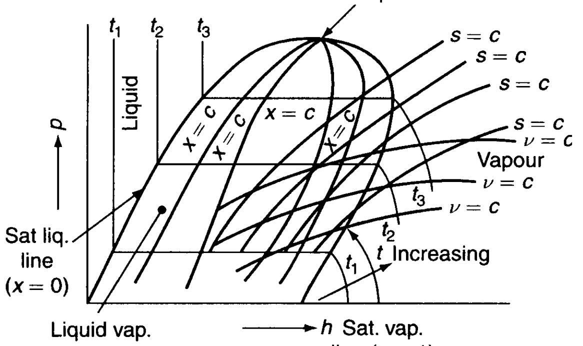


Fig. 14.6 *Phase diagram with constant property lines on p-h plot*

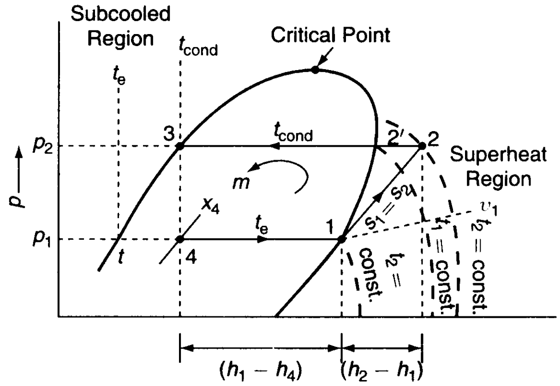


Fig. 14.7 *Vapour compression cycle on p-h diagram*

Condenser $h_2 = Q_1 + h_3$

$\therefore$ $Q_1 = (h_2 - h_3)$ kJ/kg

Expansion valve $h_3 = h_4$

or $(h_f)p_1 = (h_f)p_2 + x_4(h_{fg})p_2$

$$\therefore \quad x_4 = \frac{(h_f)p_1 - (h_f)p_2}{(h_{fg})p_2}$$

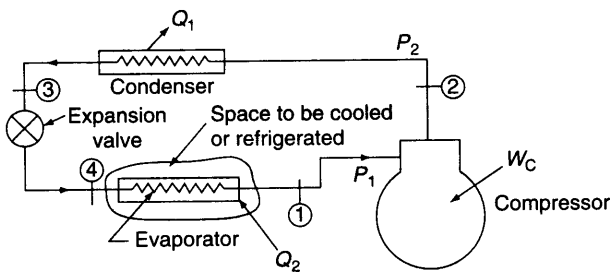


Fig. 14.8 *Vapour compression plant*

This is the quality of the refrigerant at the inlet to the evaporator (mass fraction of vapour in liquid-vapour mixture).

Evaporator $h_4 = Q_2 = h_1$

$\therefore$ $Q_2 = (h_1 - h_4)$ kJ/kg

This is known as the *refrigerating effect,* the amount of heat removed from the surroundings per unit mass flow of refrigerant.

If the *p-h* chart for a particular refrigerant is available with the given parameters, it is possible to obtain from the chart the values of enthalpy at all the cardinal points of the cycle. Then for the cycle

$$\text{COP} = \frac{Q_2}{W_c} = \frac{h_1 - h_4}{h_2 - h_1} \quad (14.2)$$

If w is the mass flow of refrigerant in kg/s, then the rate of heat removal from the surroundings

$$= w(h_1 - h_4) \text{ kJ/s} = w(h_1 - h_4) \times 3600 \text{ kJ/h}$$

One tonne of refrigeration is defined as the rate of heat removal from the surroundings equivalent to the heat required for melting 1 tonne of ice in one day. If the latent heat of fusion of ice is taken as 336 kJ/kg, then 1 tonne is equivalent to heat removal at the rate of (1000 × 336)/24 or 14,000 kJ/h

$\therefore$ Capacity of the refrigerating plant $\dfrac{w(h_1 - h_4) \times 3600}{14{,}000}$ tonnes

One ton or ton or tonne of refrigeration is SI units is often taken approximately equivalent to the heat removal rate of 3.5 kW or 210 kJ/min or 12,600 kJ/h.

The rate of heat removal in the condenser

$$Q_1 = w(h_2 - h_3) \text{ kJ/s}$$

If the condenser is water-cooled, $\dot{m}_c$ the flow-rate of cooling water in kg/s, and $(t_{c2} - t_{c1})$ the rise in temperature of water, then

$$Q_1 = w(h_2 - h_3) = \dot{m}_c c_c (t_{c2} - t_{c1}) \text{ kJ/s}$$

provided the heat transfer is confined only between the refrigerant and water, and there is no heat interaction with the surroundings.

The rate of work input to the compressor

$$W_c = w(h_2 - h_1) \text{ kJ/s}$$

Expressing the power consumption per ton of refrigeration as *unit power consumption,* we have mass flow rate and power consumption per ton refrigeration.

$$\omega = \frac{3.5}{Q_2} \text{ kg/(SATR)}$$

$$W_e = 3.5\left(\frac{h_2 - h_1}{h_1 - h_4}\right)\text{kW}/\text{TR} = \frac{3.5}{\text{COP}}\text{kW}/\text{TR}$$

Theoretical piston displacement of the compressor or suction volume flow rate

$$\dot{V} = \omega \upsilon_1 \text{ m}^3/\text{s}$$

where υ_1 is the specific volume at suction:

$$\text{Actual piston displacement} = \frac{\omega \upsilon_1}{\eta_{\text{vol}}} \text{ m}^3/\text{s}$$

where η_{vol} is the volumetric efficiency, defined later. Suction volume requirement per ton is

$$\dot{V} = \frac{210}{Q_2}\upsilon_1 \text{ m}^3/(\text{min} \times \text{TR})$$

The isentropic discharge temperature t_2 can be found by one of the following three methods:

(i) Graphically, from the p–h diagram of the refrigerant by drawing the isentropic line from point 1 (at pressure p_1) to pressure p_2 or by iteration, finding t_2 corresponding to $s_2 = s_1$.

(ii) Using saturation properties and the specific heat of vapour c_p,

$$s_1 = s_2 = s_2' + c_p \ln \frac{T_2}{T_2'}$$

(iii) Using superheat tables and interpolating for the degree of superheat $(T_2 - T_2')$ corresponding to the entropy difference $(s_2 - s_2')$ which is known.

14.3.2 Actual Vapour Compression Cycle

In order to ascertain that there is no droplet of liquid refrigerant being carried over into the compressor, some superheating of vapour is recommended after the evaporator.

A small degree of subcooling of the liquid refrigerant after the condenser is also used to reduce the mass of vapour formed during expansion, so that too many vapour bubbles do not impede the flow of liquid refrigerant through the expansion valve.

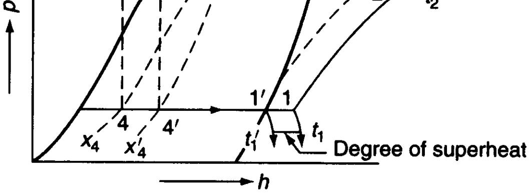


Fig. 14.9 Superheat and subcooling in a vapour compression cycle

Both the superheating of vapour at the evaporator outlet and the subcooling of liquid at the condenser outlet contribute to an increase in the refrigerating effect, as shown in Fig. 14.9. The compressor discharge temperature, however, increases, due to superheat, from t_2' to t_2, and the load on the condenser also increases.

Sometimes, a liquid-line heat exchanger is used in the plant, as shown in Fig. 14.10. The liquid is subcooled in the heat exchanger, reducing the load on the condenser and improving the COP. For 1 kg flow

$$Q_2 = h_6 - h_5,\ Q_1 = h_2 - h_3$$

$$W_c = h_2 - h_1$$

and

$$h_1 - h_6 = h_3 - h_4$$

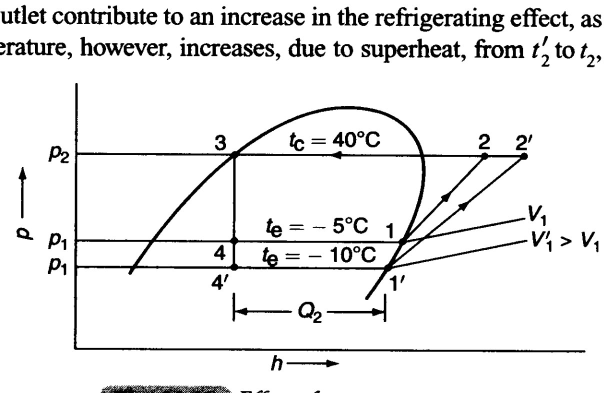


Fig. 14.10 Effect of evaporator pressure

14.3.3 Effect of Change in Operating Conditions on the Performance of Vapour Compression Cycle

(a) Effect of Evaporator Pressure For a single stage, freon compressor, let $t_{ev} = -5°C$, $t_{cond} = 40°C$. Let there be a decrease of evaporator temperature to $-10°C$. (Fig. 14.10). It is found that there is

(i) a decrease in refrigerating effect from $(h_1 - h_4)$ to $(h_1' - h_4')$

(ii) an increase in specific volume of suction vapour from v_1 to v_1' $(v_1' > v_1)$.

(iii) a decrease in volumetric efficiency, due to increase in the pressure ratio

(iv) an increase in compressor work from $(h_2 - h_1)$ to $(h_2' - h_1')$.

(b) Effect of Condenser Pressure An increase in condensing pressure results in a decrease of the refrigerating capacity and an increase in power consumption, as seen from the cycle 1–2–3–4 changed to cycle 1–2′–3′–4′ for $t_c' = 45°C$ from $t_c = 40°C$, as seen in Fig. 14.11. The decrease in refrigerating capacity is due to the decrease in refrigerating effect and volumetric efficiency. The increase in power consumption is due to the increased mass flow due to decreased refrigerating effect and an increase in specific work due to increased pressure ratio, although the isentropic line remains the same.

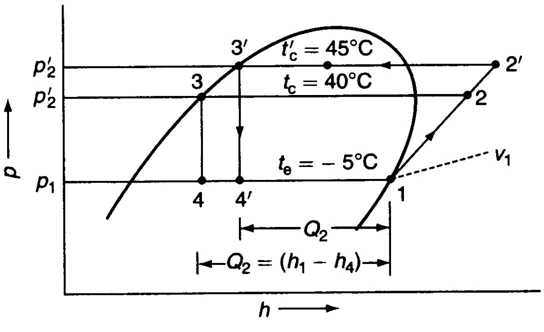


Fig. 14.11 *Effect of condenser pressure*

(c) Effect of Suction Vapour Superheat
Superheating of suction vapour is useful, as mentioned earlier, because it ensures complete vaporization of liquid in the evaporator before it enters the compressor. It can be seen from Fig. 14.12, that the effect of superheating of the vapour from $t_1 = t_e$ to t_1' is as follows:

(i) increase in specific volume of suction vapour from v_1 to v_1'

(ii) increase in refrigerating effect from $(h_1 - h_4)$ to $(h_1' - h_4)$.

(iii) increase in specific work from $(h_2 - h_1)$ to $(h_2' - h_1')$.

(iv) increase in condenser load from $(h_2 - h_3)$ to $(h_2' - h_3)$.

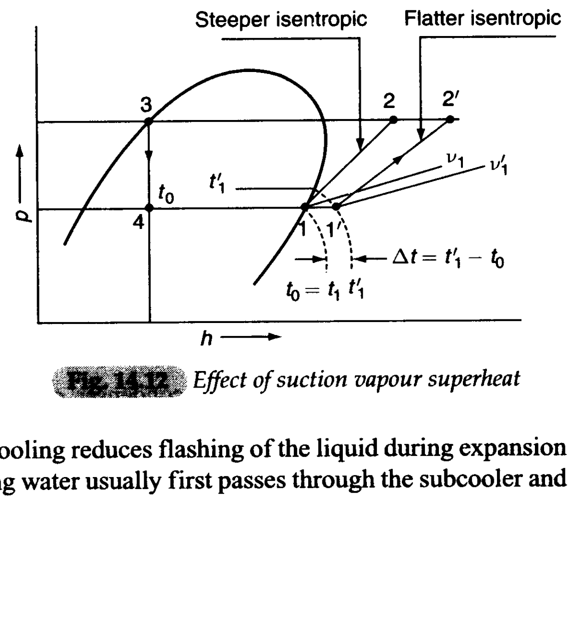


Fig. 14.12 *Effect of suction vapour superheat*

(d) Effect of Liquid Subcooling It is possible to reduce the temperature of the liquid refrigerant by installing a *subcooler* between the condenser and the expansion valve. The effect of subcooling of the liquid from $t_3 = t_4$ to t_3' (Fig. 14.13), it is seen that subcooling reduces flashing of the liquid during expansion $(x_4' < x_4)$ and increases the refrigerating effect. Cooling water usually first passes through the subcooler and

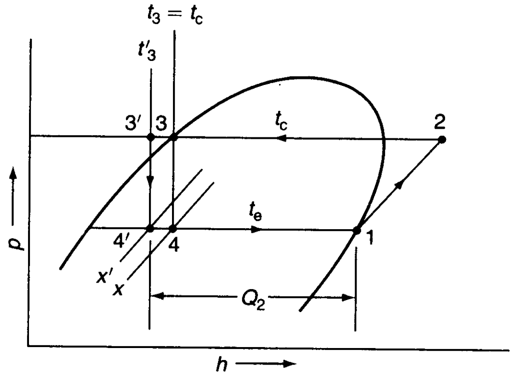


Fig. 14.13 *Effect of liquid subcooling*

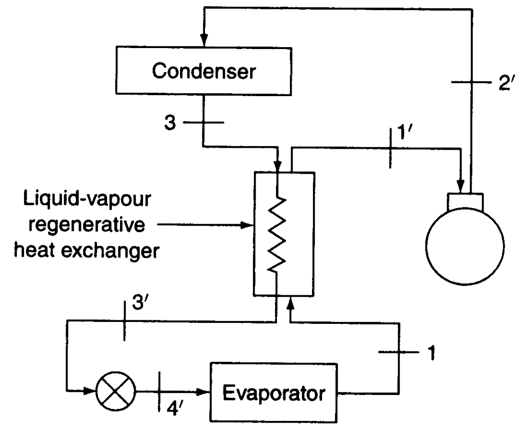


Fig. 14.14 *Effect of using suction line regenerative heat exchange*

then through the condenser. Subcooling of liquid reduces bubble formation which impedes the flow through the expansion valve or capillary tube.

(e) *Effect of Using Suction Line Regenerative Heat Exchanger* A liquid – vapour regenerative heat exchanger may be installed as shown in Fig. 14.14. In this, the refrigerant from the evaporator is superheated in the regenerator with consequent sub-cooling of the liquid from the condenser (Fig. 14.15). Since the mass flow rate of the liquid and vapour is the same, by energy balance we have

$$h_1' - h_1 = h_3 - h_3'$$

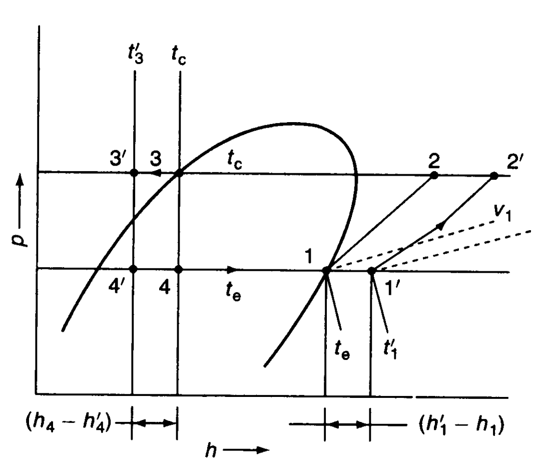


Fig. 14.15 *Vapour compression system with liquid-vapour regenerative heat exchanger*

The degree of superheat $(t_1' - t_e)$ and the degree of subcooling $(t_c - t_3')$ need not be the same, since the specific heats of the vapour and liquid phases are different. For 1-kg flow,

$$Q_2 = h_1 - h_4, \quad Q_1 = h_2' - h_3'$$

$$W_c = h_2' - h_1', \quad h_1' - h_1 = h_3 - h_3'$$

$$\text{COP} = \frac{h_1 - h_4'}{h_2' - h_1'}.$$

14.3.4 Actual Vapour Compression Cycle

As the refrigerant flows through the condenser, evaporator and piping, there will be drops in pressure. There will also be heat losses or gains depending on the temperature difference between the refrigerant and the surroundings. Further, compression will be polytropic with friction and heat transfer instead of isentropic. The actual vapour compression cycle may have some or all of the items of departure from the simple saturation cycle as enlisted below and shown on the *p-h* diagram as shown in Fig. 14.16.

(i) Superheating of vapour in the evaporator, $1d - 1c$
(ii) Heat gain and superheating of the vapour in the suction line, $1c - 1b$
(iii) Pressure drop in the suction line, $1b - 1a$
(iv) Pressure drop due to throttling in the compressor–suction valve, $1a - 1$
(v) Polytropic compression, $1 - 2$
(vi) Pressure drop at the compressor–discharge valve, $2 - 2a$
(vii) Pressure drop in the delivery line, $2a - 2b$
(viii) Heat loss and desuperheating of vapour in the delivery line, $2b - 2c$
(ix) Pressure drop in the condenser, $2b - 3$
(x) Subcooling of the liquid in the condenser, $3 - 3a$
(xi) Heat gain in the liquid line, $3a - 3b$
(xii) Pressure drop in the evaporator, $4 - 1d$

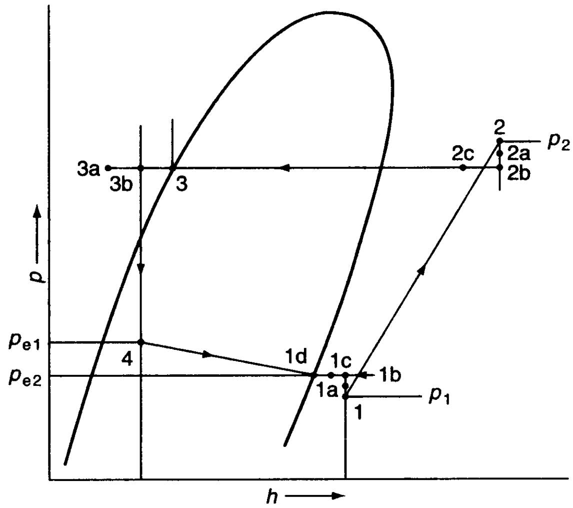


Fig. 14.16 *Actual vapour compression cycle on p-h diagram*

It may be observed that the pressure drop in the evaporator is large, while that in the condenser is not significant.

14.3.5 Components in a Vapour Compression Plant

(a) Condenser It is in the condenser that heat is rejected in a vapour compression plant. Desuperheating of the vapour takes place in the discharge line and in the first few coils of the condenser. It is followed by the condensation of the vapour at the saturation temperature. Then, subcooling may take place near the bottom. However, the sensible heat of the desuperheating and subcooling processes is quite small compared to the latent heat of condensation.

The type of condenser is generally characterized by the cooling medium used. There are three types of condensers: (i) air-cooled condensers, (ii) water-cooled condensers, and (iii) evaporative condensers. An air-cooled condenser is used in small self-contained units below 5 TR because of high power consumption and large fan noise.

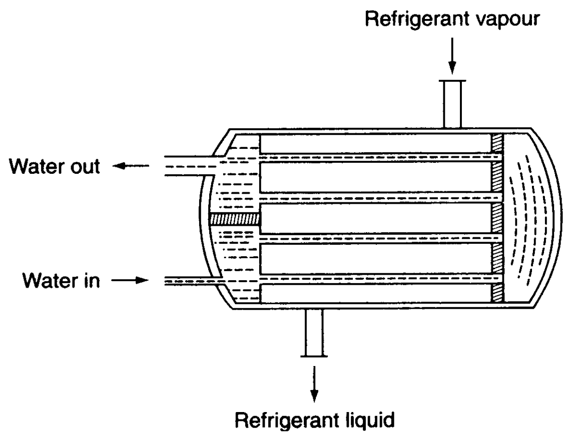


Fig. 14.17 *Schematic representation of a two-pass water-cooled shell and tube condenser*

Water-cooled condensers can be of three types, viz., shell and tube, shell and coil and double tube. The shell-and-tube type with water flowing through tubes and the refrigerant condensing in the shell is the most commonly used condenser (Fig. 14.17).

(b) Expansion Device An expansion device in a refrigeration system normally serves two functions, viz., (i) it reduces the pressure from the condenser to the evaporator, and (ii) it regulates the flow of the refrigerant to the evaporator depending on the load. It is essentially a restriction offering resistance to flow so that the pressure drops, resulting in a throttling process. There are two types of such devices, viz.,

(i) Variable restriction type
(ii) Constant restriction type

In the variable restriction type, the extent of opening or area of flow keeps on changing depending on the control required. There are two types of such control devices:

(i) Automatic expansion valve
(ii) Thermostatic expansion valve

The constant restriction type device is the *capillary tube* which is a long narrow tube of small diameter. Once the size and length are fixed, the evaporator pressure, etc., gets fixed. No modification in the operating conditions is possible.

The basic function of an automatic expansion valve (A.E.V.)* is to maintain constant pressure in the evaporator. A bellows is acted on by forces to push a needle to open or close the orifice with the variation of cooling load.

A thermostatic expansion valve, (T.E.V.) maintains a constant degree of superheat in the evaporator. A sensing bulb at the evaporator outlet maintains the constant temperature of the refrigerant entering the compressor (Fig. 14.18).

(c) Compressor Compressors may be of three types: (1) reciprocating, (b) rotary, and (c) centrifugal. When the volume flow rate of the refrigerant is large, centrifugal compressors are used. Rotary compressors are used for small units. Reciprocating compressors are used in plants up to 100 tonnes capacity. For plants of higher capacities, centrifugal compressors are employed.

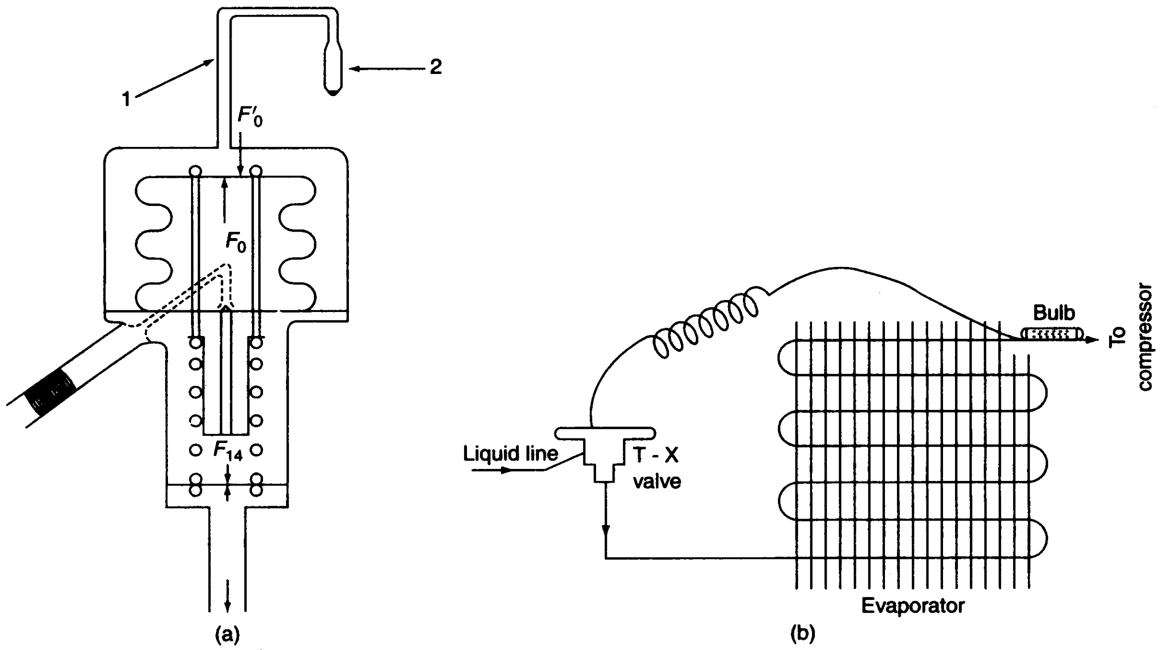


Fig. 14.18 *Arrangement showing installation of thermostatic expansion valve and its thermal bulb*

*To understand the mechanisms and functions of the A.E.V. and T.E.V. See *Refrigeration and Air Conditioning* by C.P. Arora. The T.E.V. meets the varying load requirements better than the A.E.V.

In reciprocating compressors, which may be single-cylinder or multi-cylinder ones, because of clearance, leakage past the piston and valves, and throttling effects at the suction and discharge valves, the actual volume of gas drawn into the cylinder is less than the volume displaced by the piston. This is accounted for in the term *volumetric efficiency,* which is defined as

$$\eta_{vol} = \frac{\text{Actual volume of gas drawn at evaporator pressure and temperature}}{\text{Piston displacement}}$$

∴ Volume of gas handled by the compressor

$$= w \cdot v_1 (m^3/s) = \left(\frac{\pi}{4} D^2 L \frac{N}{60} n\right) \times \eta_{vol}$$

where w is the refrigerant flow rate,

v_1 is the specific volume of the refrigerant at the compressor inlet,

D and L are the diameter and stroke of the compressor,

n is the number of cylinders in the compressor, and

N is the r.p.m.

The clearance volumetric efficiency is given by Eq. (18.13)

$$\eta_{vol} = 1 + C - C\left(\frac{p_2}{p_1}\right)^{1/n}$$

where C is the clearance. For details see the Chapter 19.

(d) Evaporator The evaporator is the component of a refrigeration system in which heat is removed from air, water, brine, etc, required to be cooled by the evaporating refrigerant. Evaporators are classified as *flooded* or *dry*. In flooded evaporators, the liquid refrigerant covers the entire heat transfer surface. A float valve is used for the expansion of the refrigerant. In dry evaporators, a part of the heat transfer surface is used for the superheating of vapour. A thermostatic expansion valve for large units or a capillary tube for small units is used in conjunction with a dry evaporator. The refrigerant flows inside the tubes in dry or direct expansion (D – X) evaporators, whereas it flows outside the tubes in flooded evaporators. One D – X coil with fins on the air side is shown in Fig. 14.19 with 4 rows of tubing and a T.E.V.

14.3.6 Multi-Stage Vapour Compression Systems

For a given condensation temperature, the lower the evaporator temperature, the higher becomes the compressor pressure ratio. For a reciprocating compressor, a high pressure ratio across a single stage means low volumetric

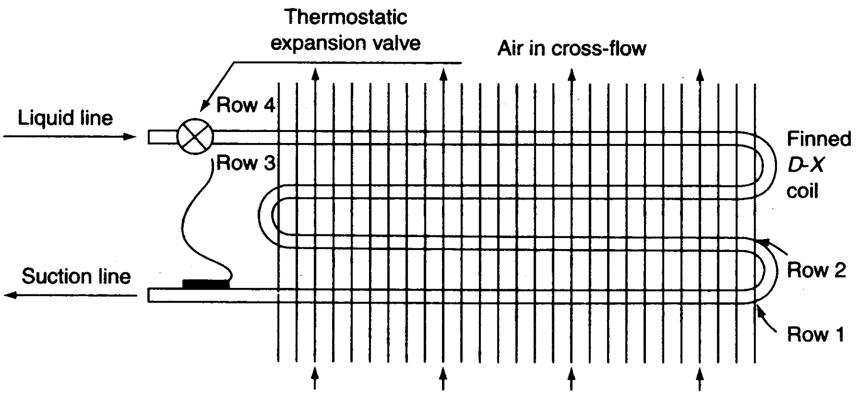


Fig. 14.19 *Illustration of a dry evaporator, direct-expansion cooling coil with thermosatic-expansion valve*

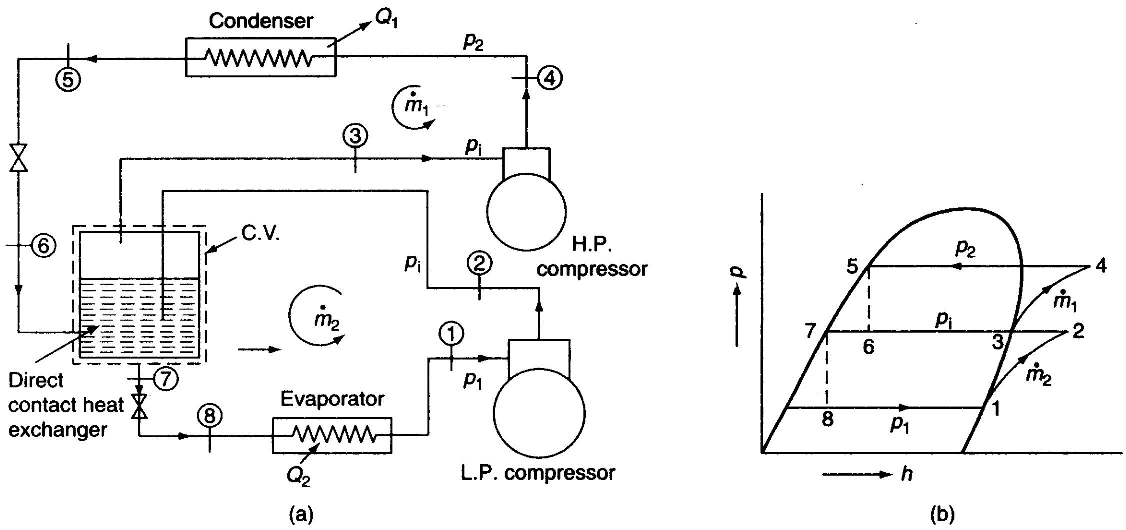


Fig. 14.20 *Two-stage vapour compression system*

efficiency. Also, with dry compression the high pressure ratio results in high compressor discharge temperature which may damage the refrigerant. To reduce the work of compression and improve the COP, multistage compression with intercooling may be adopted. Since the intercooler temperature may be below the temperature of available cooling water used for the condenser, the refrigerant itself may be used as the intercooling medium. Figure 14.20 shows a two-stage compression system with a direct contact heat exchanger.

As shown later in Sec. 14.5, for minimum work, the intercooler pressure p_i is the geometric mean of the evaporator and condenser pressures, p_1 and p_2, or

$$p_i = \sqrt{p_1 \cdot p_2}$$

However, this condition is true for complete intercooling to the initial temperature. In refrigeration systems, it is not possible. It is seen that the discharge temperature of the low-stage is much lower than that of the high stage. To reduce the discharge temperature of the higher stage, therefore the pressure ratio of the lower stage can be increased, with a corresponding reduction in the pressure ratio of the higher stage. The expression recommended for determining the intermediate pressure in refrigeration systems is

$$P_i = \sqrt{P_c P_e^{T_c/T_e}}.$$

By making an energy balance of the direct contact heat exchanger,

$$\dot{m}_2 h_2 + \dot{m}_1 h_6 = \dot{m}_2 h_7 + \dot{m}_1 h_3$$

$$\therefore \quad \frac{\dot{m}_1}{\dot{m}_2} = \frac{h_2 - h_7}{h_3 - h_6}$$

The desired refrigerating effect determines the flow rate in the low pressure loop, $\dot{m}_2$, as given below

$$\dot{m}_2 (h_1 - h_8) = \frac{14000}{3600} \times P$$

where P is the capacity, in tonnes of refrigeration.

$$\dot{m}_2 = \frac{3.89P}{h_1 - h_8} \text{kg/s}$$